On Price Regulation in the ESG Rating Industry

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Abstract. A profitable, but potentially socially harmful project is financed by an uninformed lender inclined to fund socially valuable initiatives. To make more accurate investment decisions, the lender can buy information about the project type from two (or more) competing certification providers (raters). We compare two regulatory regimes: a laissez-faire regime, where raters are free to choose prices and information precision to maximize profits, and a regulated regime where raters are subject to price oversight by a regulator that maximizes the total value generated by the project. We show that information precision monotonically increases in price. When the project revenue features a decreasing Arrow-Pratt index, raters under-invest in information precision, and a price floor restores efficiency. Conversely, raters over-invest in precision when the project revenue features an increasing Arrow-Pratt index. Then a price cap restores efficiency. No regulation is needed in the knife-edge case of a constant Arrow-Pratt index. More generally, these findings indicate that, if misplaced, an indiscriminate imposition of price controls, such as universal price caps or floors, can significantly reduce value and welfare.

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1 Introduction

In recent years, growing public concern about climate change, resource depletion, and social inequality has led consumers and investors to demand greater transparency and accountability from firms. In response, some (but not necessarily all) companies have increasingly prioritized sustainability to enhance market appeal, limit regulatory scrutiny, and mitigate environmental and social risks.

Environmental, Social, and Governance (ESG) ratings play a central role in this landscape by providing standardized, quantifiable measures of firms' 'sustainability' performance. These ratings enable investors to identify sustainability leaders and allocate capital toward firms that promote environmental protection, social responsibility, and effective governance. As such, ESG ratings have become a key instrument for stakeholders seeking to support ethical business practices and sustainable growth.

However, as an information market, the ESG rating industry faces challenges similar to those long identified in traditional credit rating markets, including conflicts of interest and limited transparency. Regulatory intervention can help address these concerns, particularly when rating agencies' revenues depend on the firms they rate. In such issuer-pay models, firms may resist paying for unfavorable ratings, creating incentives for rating inflation.

In principle, these conflicts could be mitigated by decoupling payments from rating outcomes. In practice, however, monitoring the precise contractual relationships between firms and raters is difficult. In the ESG industry, most providers have therefore adopted a subscriber-pay model, in which investors or other market participants purchase access to ratings. By shifting revenues away from rated firms, the subscriber-pay model can reduce conflicts of interest and make rating quality the primary competitive dimension, potentially improving accuracy and objectivity.

Yet two important questions remain: is moving from issuer-pay to subscriber-pay sufficient to ensure efficient ESG rating markets? Do these markets still require regulation to achieve value-maximizing outcomes, and if so, when and through which instruments?

Azarmsa and Shapiro (2024) identify one key regulatory concern: insufficient specialization among raters. They argue that welfare is maximized when providers specialize in distinct ESG dimensions, thereby increasing the total information conveyed. Competitive pressure, however, may instead induce excessive generalization, reducing informational content in equilibrium.

Our paper examines a different normative dimension. We ask whether, even absent specialization concerns, market forces can lead to over- or under-provision of information relative to what maximizes value (welfare).[1] The model features an entrepreneur seeking external financing for a profitable but potentially socially harmful project, an uninformed lender (investor) who values social impact but cannot observe project type, and two (or more) competing certification providers (raters) that acquire and sell information about the project's social value. To generate meaningful incentives, we assume limited liability and imperfect enforcement: in the event of default, author-

[1] That is, the sum of all agents' expected payoffs.

ities can seize only a fraction of realized revenues, allowing the entrepreneur to divert part of the returns as private benefits.[2]

We compare two regulatory regimes: laissez-faire and price regulation. Under laissez-faire, providers freely choose both prices and information precision. Under price regulation, a regulator sets prices to maximize total value, while providers adjust precision accordingly. This regulatory focus on prices reflects established policy practice. Policymakers have long relied on price regulation as a pragmatic tool in rating and certification markets, where information quality is difficult to observe and therefore hard to regulate directly. In financial markets, for example, oversight of credit rating agencies in both the EU and the US constrains fees through transparency, proportionality, and non-discrimination requirements, reflecting concerns that unregulated pricing may distort incentives for information provision.[3,4] Similar approaches are used in environmental certification, audit services, pharmaceutical approval, and professional licensing, where regulators frequently impose fee caps, fixed fees, or cost-based pricing rules to balance access, accuracy, and market power.[5]

In the baseline analysis, we focus on equilibria with full market coverage and single-homing, where the lender purchases information from only one provider. In these equilibria, providers charge supra-competitive prices reflecting horizontal differentiation, and choose precision by trading off information costs against increased demand from lenders who favor socially valuable projects. This tension generates a somewhat natural positive relationship between prices and precision: higher prices, and thus higher profit margins, support greater investment in information quality.

We then analyze price regulation. We first characterize the value-maximizing level of precision and compare it to the laissez-faire equilibrium. We show that the relationship between equilibrium and value-maximizing outcomes is governed by the curvature of the project's revenue function. When lenders are not overly biased — so that both project types may be financed — whether regulation calls for higher or lower precision depends on the Arrow–Pratt index of absolute risk aversion of project revenues with respect to loan size. If absolute risk aversion is decreasing (DARA),

[2] If enforcement were perfect, the entrepreneur could not divert funds and would be indifferent between investing and not investing. If ties are broken in favor of investment, our results are robust; otherwise, ratings become irrelevant.

[3] For instance, the EU's Credit Rating Agencies Regulation (EC No 1060/2009) and associated delegated acts require registered credit rating agencies to disclose and justify their pricing policies and ensure that fees are non-discriminatory and cost-related, under supervision by the European Securities and Markets Authority (ESMA). This regulatory framework constrains rating fee practices and serves as a practical precedent for fee oversight in information markets, even without explicit price caps. Proposed EU regulation of ESG rating activities similarly emphasizes transparency and supervisory authority over ESG raters' practices

[4] In the US, credit rating agencies are regulated by the Securities and Exchange Commission (SEC) as Nationally Recognized Statistical Rating Organizations (NRSROs) under the Credit Rating Agency Reform Act and the Dodd–Frank Act. While US regulation does not impose explicit price caps, it constrains rating fees indirectly through extensive disclosure, conflict-of-interest rules, and supervisory oversight of pricing policies. Rating agencies are required to publicly disclose their fee structures and to demonstrate that fees are not structured in a way that compromises rating integrity. This regulatory approach reflects the concern that fee-setting practices can affect incentives for information provision, even when accuracy itself cannot be directly regulated.

[5] Recent policy proposals for ESG rating providers in the EU follow this tradition, emphasizing fee transparency and justification as a means to prevent both rent extraction and underinvestment in rating quality. These regulatory practices underscore that price instruments are a central — and often preferred — policy lever for correcting inefficiencies in information markets, aligning closely with the mechanisms studied in this paper.

providers under-invest in precision; if it is increasing (IARA), laissez-faire yields excessive precision; if it is constant (CARA), no intervention is needed. By contrast, when lenders finance only socially valuable projects and would not invest without information, equilibrium precision is always inefficiently low.

Since equilibrium precision increases with regulated prices, inefficiencies can be corrected through price floors when precision is insufficiently low and price caps when it is excessive. Indiscriminate price regulation, however, can reduce welfare if it ignores how revenues respond to project scale. Notably, DARA — often considered empirically plausible — makes price floors particularly relevant.[6]

We then relax key assumptions to test robustness. Allowing for partial market coverage introduces a new trade-off: higher prices increase precision but reduce coverage by making uninformed lending more attractive. Similar forces arise under multi-homing, where price increases raise both the cost and the value of purchasing multiple ratings. While inframarginal lenders always benefit from higher precision, the effect on marginal lenders is ambiguous.

In a Salop (1979) model with endogenous entry, higher prices both raise profits and increase certification costs through higher precision. When cost effects dominate, a price floor helps curb excess entry; when profit effects dominate, a price cap is preferable.

Our results also extend to settings with multiple ESG dimensions, continuous social values, and enforcement that varies with project scale. Moreover, they hold when private benefits rise with project size, provided they are concave.

Finally, we contrast our findings with issuer-pay models. Under issuer-pay, providers may supply uninformative ratings when increased precision reduces entrepreneurial rents, potentially leading to market collapse. When informative ratings are supplied, precision remains inefficiently chosen because lender utility is not internalized. As a result, optimal regulation under issuer-pay always requires a price floor.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the baseline model and results. Section 4 explores extensions and robustness. Section 5 concludes. All proofs are in the Appendix.

2 Related literature

The theoretical literature on ESG ratings is still in its early stages. This lack of development is partly due to the presumption that the markets for ESG and credit ratings function similarly and exhibit comparable dynamics. The body of research on credit rating agencies (CRA) is well

[6] DARA is widely regarded as the most empirically and economically plausible specification for revenue or payoff functions. It captures the idea that as projects scale up, diversification, learning, and efficiency gains make marginal revenues less sensitive to additional investment. By contrast, CARA assumes scale-invariant responsiveness of marginal revenues, while IARA implies increasing fragility at larger scales — both of which are difficult to reconcile with typical production and investment environments. See, for example, Gollier (2001) for a theoretical and empirical discussion.

established and has already addressed a number of interesting issues related to the rating inflation phenomenon, its determinants, competitive and welfare implications — e.g., Bolton et al. (2012), Bar-Isaac and Shapiro (2013), Bouvard and Levy (2018), Piccolo (2021), and Piccolo and Shapiro (2022) among many others.

However, recent empirical studies indicate distinct differences between these two markets. Chatterji et al. (2016) first highlighted the substantial disagreement among ESG ratings from different providers—a phenomenon known as 'rating divergence'. Berg et al. (2022) further confirmed these findings and decomposed the divergence into components of scope, measurement, and weight. This disagreement poses significant challenges. It complicates the assessment of ESG performance for companies, funds, and portfolios, diminishes corporate incentives to enhance ESG performance, and obstructs the market's ability to price ESG performance effectively post-assessment.

This body of evidence has been given theoretical context by Azarmsa and Shapiro (2024), who, to the best of our knowledge, are the first to formally analyze ESG rating markets. While our paper shares a common focus with theirs, there are also important differences in our approaches. Specifically, unlike us, they focus on the dynamics of specialization among rating providers, positing that ESG raters offer ratings by category rather than a mere aggregate. Their findings suggest that specialization maximizes total welfare (value) by enhancing the quantity of information communicated, thus creating a discrepancy between the market solution when investors significantly value ESG performance and the optimal value-maximizing solution. The market tends to deliver less information through generalization, prompting the recommendation that regulators should mandate or incentivize rating specialization. Our model adds to this debate by determining the conditions under which a subscriber-pay model ensures value-maximizing information precision, and under what conditions regulatory intervention is required to achieve this goal. Therefore, our primary contribution to this literature is identifying mechanisms that could lead regulators interested in maximizing value (total welfare), to implement either a price floor to address under-investment or a price cap to curb over-investment. Interestingly, while in the multi-homing equilibrium of Azarmsa and Shapiro (2024)'s model, price regulation does not matter because raters are not horizontally differentiated, in our model, price regulation does impact the equilibrium outcome.

Our analysis also broadly builds upon and draws from the growing theoretical literature on information markets initiated by Admati and Pfleiderer (1986, 1990) and further explored by researchers such as Bergemann et al. (2018), Bergemann and Bonatti (2019), Huang, Yang, and Xiong (2018), Kastl et al. (2022), and Lizzeri (1999), among others. These complex models typically feature a single (monopolistic) information provider who gathers and sells information about the preferences of an informed agent to an uninformed principal, who then uses this information to interact with and extract surplus from the agent. Other researchers, including Balestrieri and Izmalkov (2014), Celik (2014), Koessler and Skreta (2014), Mylovanov and Tröger (2014), and Piccolo et al. (2015), adopt an informed-principal perspective, where privately informed sellers decide how much information about a product's quality to disclose to buyers.[7] A similar approach is seen in the growing literature on Bayesian persuasion, for instance in the works of Rayo and Segal (2010) and Kamenica and Gentzkow (2011), where, notably, there are no monetary transfers.

Our study emphasizes the novel normative aspects of price regulation in information markets where there is competition between providers. It highlights the impact of regulatory intervention in a variety of intriguing scenarios, thus deepening our understanding of the effective governance of ESG rating practices and information markets more broadly.

3 The baseline model

In this section, we present a stylized model that illustrates the fundamental trade-offs that a public authority encounters when regulating certification providers who gather and sell information about the 'social value' of a business project.

Players and the environment. A risk-neutral entrepreneur (E) needs external capital ($x \geq 0$) to fund a project that generates revenues described by the function $f(x)$, which is increasing, $f'(x) > 0$, and (weakly) concave, $f''(x) \leq 0$.

With probability $\frac{1}{2}$, the project is harmful to society; otherwise, it is socially valuable. The actual impact of the project — i.e., whether socially harmful or valuable — is private information to E. This variability is modeled with a binary random variable $\theta \in \{0, 1\}$. By convention, a project with $\theta = 0$ is classified as socially harmful, whereas a project with $\theta = 1$ is considered socially valuable. A single ESG dimension, reflects the preference of some investors for an aggregate measure of a project's ESG attributes.[8]

The type of the project does not impact its profitability[9], but does influence the preferences of the representative lender (L), whose utility function is

$$u(x) \triangleq R(x) - x + \mu\theta x.$$

The function $R(x)$, which we specify below, is the repayment that L obtains from E in exchange of a loan of size x. The parameter $\mu \in [0, 1)$ reflects the incremental value that L assigns to the

[7] For a dynamic model, see Hörner and Skrzypacz (2016).

[8] In the online Appendix, we consider both a version of the model where the project type is a continuous random variable, meaning that its impact on society is not discrete but may vary in extent, and a scenario where the project features multiple ESG attributes.

[9] Assuming that the social value of a project is uncorrelated with its revenues seems a neutral approach for several reasons. Although there is a growing demand for ethically-produced goods and services, characteristics such as sustainability practices, fair labor conditions, and robust governance often involve up-front costs and may not immediately translate into financial gains. These practices are typically designed for long-term benefits, such as sustained company reputation, reduced regulatory risks, and increased consumer loyalty, which may not immediately increase short-term revenue. Hence, whether or not ethical projects are likely to generate higher or lower revenues than traditional profit-oriented projects depends on several factors, including consumer responsiveness to these characteristics, which may vary across industries and countries, the time horizon of the project, and the regulatory regime.

socially valuable project (or, alternatively, the cost savings when the project is of type 1 instead of type 0).

For simplicity, we assume that E does not care about the social impact of the project.[10] Lenders generally care more than entrepreneurs about the social value of a project because their financial exposure is not limited to the direct returns from the project, but also includes the broader societal impacts that could influence their risk profile. Entrepreneurs are typically focused on the immediate financial outcomes and their personal stake in the project's success.[11]

Information providers. To increase the precision of its investment decisions, L can purchase information about the project type from two certification providers (raters) that are located at the end-points of a Hotelling line of length 1. The providers (P_0 and P_1) operate a *subscriber-pay* business model.[12] Without loss of generality, P_0 is located at 0, while P_1 is located at 1. L's location is uniformly distributed along the Hotelling line. When located at $z \in (0, 1)$, L pays a quadratic transport cost tz^2 if it patronizes P_0 and $t(1-z)^2$ if it patronizes P_1, where $t > 0$ is the unit transport cost or the degree of horizontal differentiation between providers — i.e., a larger t implies that the certification market is (ceteris paribus) less competitive as L is relatively less willing to switch from one provider to the other.[13]

This hypothesis implies that certification providers within a market are not perceived as identical, but rather as differentiated by investors based on the value of long-term relationships — e.g., they may trust or prefer certain providers due to established rapport, familiarity, or prior positive experiences, making them more inclined to seek ratings or certifications from these providers rather than from others with whom they have less experience.

Certification providers initially do not have information about the project type but can invest resources to learn it. Specifically, P_i observes a signal $s_i \in \{0, 1\}$ about the project type. The precision (accuracy) $\alpha_i \in [\frac{1}{2}, 1]$ of s_i is an endogenous variable. This signal structure can be

[10] Assuming that E also cares about the social impact of the project, but to a lesser extent than L, does not alter the qualitative insights of our results.

[11] If a project has negative social implications, such as environmental damage or social harm, it could lead to increased regulatory scrutiny, reputational risks, or even future liabilities, all of which affect the lender's risk. Therefore, lenders tend to have a more vested interest in ensuring that the project not only generates profit but also aligns with societal interests, as this can influence the long-term financial health of their investments. Entrepreneurs, on the other hand, may prioritize profitability over the broader social impacts if they are not directly accountable for these consequences.

[12] In the Extensions, we also consider an *issuer-pay* model, where the entrepreneur pays the providers to acquire and disclose information about the project type.

[13] We assume quadratic transport costs to ensure interior solutions, which are not always guaranteed under linear transport costs when the market is not fully covered. For consistency, we maintain this assumption in both the baseline model with full market coverage — where results are invariant to whether transport costs are linear or quadratic — and its extension to a partially covered market, where linear transport costs may instead lead to existence issues, as deviations that attract the entire market are easier to implement.

represented with the following matrix

	$s_i = 1$	$s_i = 0$
$\theta = 1$	α_i	$1 - \alpha_i$
$\theta = 0$	$1 - \alpha_i$	α_i

When $\alpha_i = \frac{1}{2}$, the signal is uninformative. When $\alpha_i = 1$, the signal is fully informative. The cost of achieving precision α_i is $c(\alpha_i)$. This cost is increasing, $c'(\alpha_i) > 0$, sufficiently convex, $c''(\alpha_i) \geq k > 0$ for every $\alpha_i \in \left[\frac{1}{2}, 1\right]$ with k large enough, and satisfies the standard Inada conditions

$$\lim_{\alpha_i \to \frac{1}{2}} c(\alpha_i) = \lim_{\alpha_i \to \frac{1}{2}} c'(\alpha_i) = 0, \qquad \lim_{\alpha_i \to 1} c'(\alpha_i) = \bar{c},$$

with $\bar{c}$ being sufficiently large. Using Bayes' rule, the posterior probabilities are

$$\Pr[\theta | s_i = \theta, \alpha_i] = \alpha_i \geq \Pr[\theta | s_i \neq \theta, \alpha_i] = 1 - \alpha_i.$$

The price that P_i charges for its service is $p_i \geq 0$. To attract L, each provider P_i offers an information policy $\wp_i \triangleq (\alpha_i, p_i)$.

Timing. The timing of the game is as follows.

$\tau = 0$ Providers offer their policies.

$\tau = 1$ L selects a provider, observes a signal realization, updates beliefs, and invests accordingly.

$\tau = 2$ Project revenues materialize.

As in Azarmsa and Shapiro (2024) and, more broadly, in the literature on information markets (e.g., Bergemann et al., 2015, and Kastl et al., 2018, among many others), we assume that providers cannot falsify signals or manipulate the precision announced in their policy.

Regulatory environment. We examine and compare two distinct regulatory environments:

- *Laissez-faire.* In this regime, providers have the autonomy to set both prices and information precision without regulatory oversight.
- *Price-regulation.* In this regime, providers are subject to price control mandated by regulation. Yet, they can adjust information precision in response to the price constraints imposed by the regulator.

Limited enforcement. To make the problem interesting, we assume that E is protected by limited liability and appropriates a fraction $\phi \in [0, 1]$ of the project revenues as private benefits.

That is, due to limited enforcement, L can seize at most $(1-\phi)\,f(x)$ of the realized revenues in the event of default.[14] The share of seizeable revenues $1-\phi$ can be interpreted as the probability of successful law enforcement, which is standardized at the industry level and generally independent of the specifics of individual default events.[15]

Since the financial agreement between E and L is not pivotal to the core insights we wish to convey, we assume that L always gets the highest repayment $R(x) = (1-\phi)\,f(x)$. This may arise either from an un-modeled competitive mechanism among entrepreneurs or because L sets the interest rate sufficiently high to induce default, thereby extracting the entire seizable revenue.[16]

Equilibrium concept. Since E is privately informed about the project's type and the game structure is sequential, the appropriate solution concept is Perfect Bayesian Equilibrium (PBE). However, since E makes no decisions, this reduces to Subgame Perfect Nash Equilibrium (SPNE). In the baseline analysis, we focus on a symmetric equilibrium with full market coverage such that both providers offer a policy $\wp^\star \triangleq (p^\star, \alpha^\star)$ and L buys information from one provider (single-homing).[17]

Payoffs. Before entering the technical details of the equilibrium analysis, it is useful to characterize the players' payoffs. Conditional on having observed signal $s_i \in \{0,1\}$, L solves the following maximization problem

$$\max_{x\geq 0}\,(1-\phi)\,f(x) - x + \mu\mathbb{E}\left[\theta|s_i,\alpha_i\right]x,$$

for which the first-order condition is

$$(1-\phi)\,f'(x) - 1 + \mu\mathbb{E}\left[\theta|s_i,\alpha_i\right] = 0 \quad\Rightarrow\quad x^\star(s_i,\alpha_i) \triangleq \varphi\left(\frac{1-\mu\mathbb{E}\left[\theta|s_i,\alpha_i\right]}{1-\phi}\right),$$

with $\varphi(\cdot)$ being defined as the inverse of the project's marginal revenue $f'(\cdot)$.

Essentially, when choosing how much to invest in the project, L behaves as a monopolist and trades off marginal revenue against the investment cost and the expected benefit associated with the project being of type 1.

[14] Limited enforcement is a typical problem in finance, irrespective of the social value of a project, because financial contracts often depend on a mix of legal frameworks, monitoring mechanisms, and enforcement tools that may not be fully effective.

[15] In the Online Appendix, we also examine the case where the loan size, x, influences the amount of revenues that can be seized in case of default, and show that our main results remain qualitatively unchanged.

[16] Notice that this formulation has implications equivalent to equity financing, in which L acquires a share $1-\phi$ of the project revenue. Hence, the repayment structure need not be interpreted literally as debt, but can also be viewed as a form of equity financing.

[17] In the Extensions, we discuss two additional classes of equilibria: (i) equilibria with partial market coverage, and (ii) equilibria with multi-homing.

L's expected payoff when it is located at $z \in (0,1)$ and patronizes P_i is, therefore,

$$u(\wp_i) \triangleq \sum_{\theta} \Pr[\theta] \sum_{s_i} \Pr[s_i|\theta] \left[(1-\phi) f(x^\star(s_i,\alpha_i)) - x^\star(s_i,\alpha_i)(1-\mu\theta)\right] - p_i - td_i$$
$$= \underbrace{\frac{1}{2}\sum_{s_i}\left[(1-\phi) f(x^\star(s_i,\alpha_i)) - x^\star(s_i,\alpha_i)\right] + \frac{\mu}{2}\sum_{s_i}\Pr[\theta=1|s_i]\, x^\star(s_i,\alpha_i)}_{\triangleq \Lambda(\alpha_i)} - p_i - td_i$$

with $d_i = z^2$ if $i = 0$ and $d_i = (1-z)^2$ if $i = 1$.

Letting $q_i(\wp_0, \wp_1) \in (0,1)$ be the probability that L patronizes P_i, the expected profit of P_i is

$$\pi_i(\wp_i) \triangleq q_i(\wp_0,\wp_1)\, p_i - c(\alpha_i).$$

That is, the difference between the expected revenue and the cost of providing precision α_i.

The project's expected value (total welfare) is measured as the sum of all players' expected payoffs.[18] The price of the rating is a pure transfer, so it is welfare-neutral and does not enter the formula. Assuming that the market is fully covered, that each provider is patronized with probability $q_i(\wp_0, \wp_1)$, and that the indifferent lender between P_0 and P_1 is located at $z(\wp_0, \wp_1) \in (0,1)$, the expected value (total welfare) is

$$\mathcal{V}(\alpha) \triangleq \underbrace{\sum_{\theta=0,1}\Pr[\theta]\sum_{i=0,1} q_i(\wp_0,\wp_1)\sum_{s_i=0,1}\Pr[s_i|\theta,\alpha_i]\left[(1-\phi) f(x^\star(s_i,\alpha_i)) - x^\star(s_i,\alpha_i)(1-\mu\theta)\right]}_{L\text{'s (expected) utility before prices and transport costs}}$$
$$+ \underbrace{\sum_{\theta=0,1}\Pr[\theta]\sum_{i=0,1} q_i(\wp_0,\wp_1)\sum_{s_i=0,1}\Pr[s_i|\theta,\alpha_i]\,\phi f(x^\star(s_i,\alpha_i))}_{E\text{'s (expected) repayment}}$$
$$\underbrace{-\sum_{i=0,1} c(\alpha_i)}_{\text{Information costs}} \underbrace{- t\left[\int_0^{z(\wp_0,\wp_1)} z^2 dz + \int_{z(\wp_0,\wp_1)}^1 (1-z)^2 dz\right]}_{\text{Transport costs}}.$$

In a symmetric equilibrium, where both providers offer the same information policy, each provider is patronized with probability $\frac{1}{2}$ and the transport cost is is $\frac{t}{12}$.

Technical assumptions. As a technical requirement, we assume that $f(\cdot)$ has bounded third-order derivatives, ensuring the existence of equilibria with interior solutions both in the *laissez-faire*

[18] We do not consider any additional social costs related to implementing a socially harmful project. Introducing such costs would naturally shift value maximization toward requiring more accuracy than what is present in equilibrium, potentially skewing the results toward a price floor. This would occur because, to mitigate social harm, regulators would prioritize higher accuracy compared to information providers, which would, in turn, lead value maximization to set higher prices than those emerging at equilibrium.

regime and under regulated prices (see the Appendix). We assume that L prefers to give a positive loan amount to both project types — i.e.,

$$(1-\phi)f'(0)-1>0 \quad \Rightarrow \quad \phi<\overline{\phi}\triangleq 1-\frac{1}{f'(0)},$$

with $f'(0)>1$. At the end of the analysis, we discuss what would happen when L is unwilling to fund a project that is known with certainty to be of type $\theta=0$.

The no-certification benchmark. Suppose that there are no certification providers. L bases its investment decisions on the prior only, offers a loan of size

$$\hat{x}\triangleq x^{\star}(0,\frac{1}{2})=x^{\star}(1,\frac{1}{2}),$$

and earns a payoff

$$\hat{u}\triangleq(1-\phi)\,f\,(\hat{x})-(1-\mu\mathbb{E}\,[\theta])\,\hat{x}.$$

The hypothesis that $\phi<\overline{\phi}$, together with the above Inada conditions, guarantee that $\hat{u}\geq 0$. In a remark, at the end of the analysis, we explain how results change when $\hat{u}<0$.

The perfect-competition benchmark. Under perfect competition ($t=0$), it is evident that prices are zero, and certification signals become uninformative, as providers would have no incentive to differentiate their services: an underinvestment problem. To increase accuracy, the regulator would then impose a price floor, stimulating competition based on accuracy rather than price.

Therefore, in the remainder of the paper, we assume that t is sufficiently large to ensure that accuracy remains positive so to capture both the case of under- and overinvestment in accuracy. Notably, the case of monopoly arises when t is sufficiently high, which is explored in the Extensions where we consider equilibria in which the market is not fully covered.

3.1 Single-homing equilibrium with full market coverage

We first determine some intuitive properties of L's investment decision.

Lemma 1 *L's optimal investment is such that*

$$x^{\star}\,(1,\alpha_i)\geq x^{\star}\,(0,\alpha_i)>0,$$

with equality only at $\alpha_i=\frac{1}{2}$. Furthermore, $x^{\star}\,(1,\alpha_i)$ is increasing in α_i, and $x^{\star}\,(0,\alpha_i)$ is decreasing in α_i.

As intuition suggests, L is more inclined to finance the project when it receives positive news about its type, and the more precise the rating, the greater its effect.[19]

[19] In the Appendix, we determine the conditions under which purchasing both ratings is not optimal.

We can now proceed to characterize L's demand for certification. Suppose that the providers offer policies $\wp_0 = (\alpha_0, p_0)$ and $\wp_1 = (\alpha_1, p_1)$. Let

$$\lambda(s_i, \alpha_i) \triangleq (1-\phi) f(x^\star(s_i, \alpha_i)) - x^\star(s_i, \alpha_i) + \mu \Pr[\theta = 1 | s_i, \alpha_i] \, x^\star(s_i, \alpha_i).$$

L is indifferent between the two providers when it is located at

$$z(\wp_0, \wp_1) \triangleq \frac{1}{2} + \frac{p_1 - p_0}{2t} + \frac{\sum_{s_0} \lambda(s_0, \alpha_0) - \sum_{s_1} \lambda(s_1, \alpha_1)}{4t},$$

which yields the probability with which P_0 is patronized. Clearly, with full market coverage, $1 - z(\wp_0, \wp_1)$ is the probability with which L patronizes P_1.

Equilibrium with laissez-faire. Each provider devises its certification policy to maximize expected profits, assuming that its competitor adheres to the equilibrium policy $\wp^\star$. Focus on P_0 without loss of generality. It solves the following maximization problem

$$\max_{\wp_0 \in \mathbb{R}_+ \times \left[\frac{1}{2}, 1\right]} z(\wp_0, \wp^\star) \, p_0 - c(\alpha_0).$$

In an interior solution, the first-order condition with respect to p_0 is

$$\underbrace{\frac{\partial z(\wp_0, \wp^\star)}{\partial p_0} p_1}_{\text{Volume effect } (-)} + \underbrace{z(\wp_0, \wp^\star)}_{\text{Margin effect } (+)} = 0. \tag{1}$$

with

$$\frac{\partial z(\wp_0, \wp^\star)}{\partial p_0} = -\frac{1}{2t} < 0.$$

Equation (1) reflects the standard trade-off between volume and profit margin. Conditional on the probability of being patronized, increasing p_0 yields P_0 a higher profit margin. However, a higher p_0 also induces a lower probability of being patronized. In a symmetric equilibrium, this trade-off yields the standard Hotelling pricing rule $p^\star = t$.

The first-order condition with respect to α_0 is

$$\frac{\partial z(\wp_0, \wp^\star)}{\partial \alpha_0} p_0 - c'(\alpha_0) = 0, \tag{2}$$

where, by the Envelope Theorem, we have

$$\frac{\partial z(\wp_0, \wp^\star)}{\partial \alpha_0} = \mu \frac{x^\star(1, \alpha_0) - x^\star(0, \alpha_0)}{4t} > 0.$$

All else being equal, policies securing higher precision are more likely to attract L, because its utility function is increasing with the project type.

For a given price p_0, let $\hat{\alpha}(p_0)$ be the solution in α_0 of (2). The following then holds

Lemma 2 *$\hat{\alpha}(p_0)$ is increasing in p_0, and is such that $\lim_{p_0 \to 0} \hat{\alpha}(p_0) = \frac{1}{2}$.*

Since demand for certification increases with precision, a higher margin (represented by p_0 since marginal costs have been normalized to zero) encourages providers to offer greater signal precision — i.e., price and precision are complements.

Substituting $p^{\star} = t$ into (2), we can establish the following.

Proposition 1 *Absent regulatory oversight, in a symmetric equilibrium with single-homing and full market coverage, both providers charge $p^{\star} = t$ and supply information precision $\alpha^{\star} \in (0.5, 1)$ that is the unique solution of*

$$\mu \frac{x^{\star}(1, \alpha^{\star}) - x^{\star}(0, \alpha^{\star})}{4} = c'(\alpha^{\star}),$$

with $\alpha^{\star}$ being increasing in μ and decreasing in ϕ.

As intuition suggests, in an equilibrium with full market coverage and single-homing, both providers price according to the standard Hotelling rule, resulting in higher prices as the degree of differentiation between them increases. Furthermore, they provide informative signals in equilibrium.[20] The precision of these signals increases with the share of profits $1 - \phi$ that L can claim in the event of default. Clearly, this precision intensifies with the weight μ that L assigns to the project type. In words, the greater the importance that L assigns to the project type, the more it is willing to pay for precision, thereby inducing providers to supply more accurate ratings.

Price regulation. Suppose now that providers' price is set by a regulator whose objective is value maximization. Since prices are just a monetary transfer, in a symmetric equilibrium with full market coverage, expected total welfare depends only on the precision set by providers. Let p be the price mandated by the regulator. Both providers set the same level of precision $\hat{\alpha}(p)$ that solves

$$\mu \frac{x^{\star}(1, \alpha) - x^{\star}(0, \alpha)}{4t} p = c'(\alpha),$$

with $\hat{\alpha}(p)$ being increasing in p.

The monotonic relationship between precision and price allows us to simplify the regulator's problem and proceed as if it could directly set α.[21] Notice that transport costs do not depend on α

[20] In the proof of the proposition, we derive sufficient conditions for the existence of the equilibrium.

[21] Having characterized the level of precision that maximizes the project's expected value, say $\alpha^{\star\star}$, we will then recover the optimal regulated price as

$$p^{\star\star} = \hat{\alpha}^{-1}(\alpha^{\star\star}) \triangleq \frac{4tc'(\alpha^{\star\star}))}{\mu[x^{\star}(1, \alpha^{\star\star}) - x^{\star}(0, \alpha^{\star\star})]}.$$

with full market coverage. Hence, maximizing the value created by the investment requires solving

$$\max_{\alpha\in\left[\frac{1}{2},1\right]}\mathcal{V}(\alpha)\triangleq\max_{\alpha\in\left[\frac{1}{2},1\right]}\frac{1}{2}\sum_{s=0,1}\left[f\left(x^{\star}(s,\alpha)\right)-x^{\star}(s,\alpha)\right]+\frac{\mu}{2}\sum_{s=0,1}\Pr\left[s|\theta=1\right]x^{\star}(s,\alpha)-2c(\alpha),$$

Essentially, the value created by the investment can be decomposed into three intuitive components: the monetary surplus generated by the project (revenue net of the investment cost), the effect of the project type on L's utility, and the cost of gathering information.

Notably, in contrast to the equilibrium analysis, the regulator internalizes E's utility. Therefore, the level of precision that maximizes value also accounts for the impact of greater precision on E's utility across the different realizations of the ratings — i.e., a larger α reduces E's profit in state $s=0$ and increases it in state $s=1$.

To gain insights on the forces that shape value maximization, define

$$\zeta(x)\triangleq-\frac{d\ln f'(x)}{dx}=-\frac{f''(x)}{f'(x)}\geq 0.$$

This ratio corresponds to the Arrow–Pratt index of absolute risk aversion of the project revenue function, and measures the curvature of this function. The derivative of this index

$$\zeta'(x)=-\frac{d^2\ln f'(x)}{dx^2},$$

measures how the curvature of the revenue function changes with the scale of the project. A negative derivative ($\zeta'(x)<0$ for all $x\geq 0$) corresponds to decreasing absolute risk aversion (DARA): marginal revenue becomes less sensitive to project scale as the scale increases. A positive derivative ($\zeta'(x)>0$ for all $x\geq 0$) corresponds to increasing absolute risk aversion (IARA): marginal revenue becomes more sensitive to scale at higher levels. When $\zeta'(x)=0$ for all $x\geq 0$, absolute risk aversion is constant (CARA), implying that marginal revenue declines at a constant rate.

By the Envelope Theorem, we obtain:

$$\mathcal{V}'(\alpha)=\mu\frac{x^{\star}(1,\alpha)-x^{\star}(0,\alpha)}{2}-2c'(\alpha)+\frac{\phi\mu}{2(1-\phi)}\left[\frac{1}{\zeta\left(x^{\star}(1,\alpha)\right)}-\frac{1}{\zeta\left(x^{\star}(0,\alpha)\right)}\right].\tag{3}$$

Let $\alpha^{\star\star}$ be the unique solution of $\mathcal{V}'(\alpha)=0$.[22] The following holds.

Proposition 2 *Suppose that $\phi>0$. Then, if $\zeta'(\cdot)<0$ for all $x\geq 0$, the level of precision that maximizes value is higher than the level of precision that the providers set in the laissez-faire equilibrium — i.e., $\alpha^{\star\star}>\alpha^{\star}$. By contrast, if $\zeta'(\cdot)>0$ for all $x\geq 0$ the level of precision that maximizes the project's value is lower than the level of precision that the providers set in the laissez-faire equilibrium — i.e., $\alpha^{\star\star}<\alpha^{\star}$. The optimal regulated pice is $p^{\star\star}=\hat{\alpha}^{-1}(\alpha^{\star\star})$.*

[22] In the Appendix, we derive sufficient conditions for $\mathcal{V}''(\alpha)<0$.

If, instead, $\phi = 0$ or $\zeta'(\cdot) = 0$ for all $x \geq 0$, the level of precision that maximizes value is the same as the level of precision set in the laissez-faire equilibrium — i.e., $\alpha^{\star\star} = \alpha^{\star}$, and thus $p^{\star\star} = p^{\star}$.

This proposition demonstrates that whether value maximization mandates a higher or lower level of precision than that emerging in an equilibrium with laissez-faire depends on how rapidly the marginal revenue varies with the scale of the loan. When the marginal revenue generated by the project becomes less responsive to an unit increase in capital at higher scales — e.g., because of limited scale economies — providers under-invest in precision compared to what value maximization would mandate. Conversely, value maximization calls for lower precision than that emerging under laissez-faire when the marginal revenue is more responsive to a unit increase in capital at higher scales — e.g., because of strong scale economies.

The intuition is that providers do not fully internalize the effect of their investment in accuracy on L's expected utility. Greater accuracy increases investment in state $s = 1$ and reduces it in state $s = 0$. How these changes affect L's expected utility depends on how the curvature of the revenue function varies with project scale, namely, on whether revenues respond more strongly to the increase in the loan in state $s = 1$ than to the reduction in the loan in state $s = 0$. When $\zeta'(\cdot) < 0$ for every $x \geq 0$, by increasing the loan in state $s = 1$, increased precision mitigates the extent of diminishing returns and enhances the revenue of a type-1 project more than it reduces that of a type-0 project. Conversely, when $\zeta'(\cdot) > 0$ for every $x \geq 0$, increased precision disproportionately penalizes a type-0 project relative to the benefits it brings to a type-1 project.

The knife-edge case arises when the revenue function has a constant concavity index, $\zeta'(x) = 0$ for every $x \geq 0$. In this case, the degree of decreasing returns to scale is invariant to loan size, so the effect of increased precision on the revenues of the two project types exactly offsets. Hence, the value-maximizing level of precision coincides with the equilibrium level of precision, and no regulation is needed.

The next proposition shows how the discrepancy between the precision chosen by the regulator and that arising in a laissez-faire equilibrium is reflected in the optimal regulated price.

Proposition 3 *Suppose that $\phi > 0$. Value maximization requires a binding price floor (resp. a price cap) if $\zeta'(\cdot) < 0$ for all $x \geq 0$ (resp. $>$). By contrast, if $\zeta(\cdot)$ is constant or $\phi = 0$, the equilibrium with laissez-faire maximizes expected value, so price regulation is unnecessary.*

The difference $p^{\star\star} - p^{\star}$ is increasing in t when $p^{\star\star} > p^{\star}$ and decreasing in t when $p^{\star\star} < p^{\star}$ — i.e., market power magnifies the need for regulatory intervention regardless of the type of intervention.

In words, when the market equilibrium exhibits an under-supply of precision, the regulator can correct this inefficiency by imposing an appropriately chosen price floor. By contrast, when providers supply excessive precision in equilibrium, an appropriately designed price cap restores efficiency. As a result, the indiscriminate use of price caps or price floors may reduce welfare in the absence of information about the concavity of the project's revenue function — or, more concretely, about the nature and magnitude of the associated scale economies.

Moreover, an increase in product differentiation — that is, stronger market power — tends to widen the gap between value-maximizing and equilibrium outcomes. Greater market power reduces the extent to which providers internalize E's welfare in their decisions, thereby amplifying the inefficiency identified above.

Remark. So far, we have assumed that L invests at the prior since $\hat{u} \geq 0$ — i.e., absent information, it prefers to finance the project. Suppose now that this is no longer the case because ϕ is sufficiently large. This implies, *a fortiori*, that L does not invest upon receiving bad news ($s_i = 0$). In this case, it is straightforward to show that a price floor is unambiguously optimal from a value-maximization perspective, since $x^{\star}(0, \alpha) = 0$.

4 Extensions

To test the robustness of the baseline results and explore additional aspects of ESG ratings regulation, this section relaxes several key assumptions of the model and extends it along multiple dimensions. In particular, we consider partially covered markets, the possibility that L purchases multiple ratings, and competition among multiple providers with endogenous entry. To further relate our findings to the traditional rating-agency literature, we also introduce an issuer-pay model in which it is E rather than L, who pays for the rating.

Additional extensions — including projects with multiple dimensions, a continuous type space, and enforcement regimes that depend on project scale — are developed in the online Appendix.

4.1 Equilibria with a partially covered market

In this section, we consider single-homing equilibria with partial market coverage — i.e., the case where there are two local monopolies at the extremes of the Hotelling line. In such an equilibrium, L decides with positive probability not to purchase information.[23]

Consider a symmetric equilibrium in which both providers offer a policy $\wp^{\star} \triangleq (p^{\star}, \alpha^{\star})$, but assume that when L is located in the interval $(z^{\star}, 1 - z^{\star})$, with $z^{\star} \in \left(0, \frac{1}{2}\right)$, it decides not to buy information and invests based only on the prior. A lender located below the threshold $z^{\star}$ buys $\wp^{\star}$ from P_0, and a lender located above $1 - z^{\star}$ buys from P_1. As before, focus on P_0 and assume that P_1 sticks to the equilibrium.

[23] Of course, we need to ensure that when L prefers not to be informed rather than buying only one rating, it is also unwilling to buy both ratings (see the Appendix). Intuitively, this is easy, because the marginal informativeness of the second rating is less than that of the first, both ratings cost the same in a symmetric equilibrium, and L first buys from the closer ratings provider, so buying the second rating requires a greater transport cost.

A lender who patronizes provider P_0 obtains expected utility

$$u^{\star}(\wp_0) \triangleq \Lambda(\alpha_0) - p_0 - tz^2,$$

with

$$\Lambda(\alpha_0) \triangleq \frac{1}{2} \sum_{s_0=0,1} \lambda(s_0, \alpha_0).$$

The location at which L is indifferent between patronizing P_0 and not buying a rating is

$$z^{\star}(\wp_0) \triangleq \sqrt{\frac{\Lambda(\alpha_0) - \hat{u} - p_0}{t}},$$

which is also the probability with which L patronizes provider P_0. Hence, P_0's maximization problem is

$$\max_{\wp_0 \in \mathbb{R}_+ \times \left[\frac{1}{2}, 1\right]} z^{\star}(\wp_0) p_0 - c(\alpha_0).$$

In an interior solution, the first-order conditions with respect to p_0 and α_0 yield, respectively,

$$p_0(\alpha_0) = \frac{2}{3}(\Lambda(\alpha_0) - \hat{u}), \tag{4}$$

$$\mu \frac{x^{\star}(1, \alpha_0) - x^{\star}(0, \alpha_0)}{2} \frac{p_0}{2\sqrt{t}\sqrt{\Lambda(\alpha_0) - \hat{u} - p_0}} - c'(\alpha_0) = 0, \tag{5}$$

which mirrors the trade-off identified in the baseline model, with the caveat that here the market is not fully covered — i.e., an increase in p_0, expands the region of parameters where L does not purchase information. Let $\alpha_0(p_0)$ be the solution of (5). Since $\Lambda(\alpha_0) > \hat{u}$ for every $\alpha_0 > \frac{1}{2}$, the following holds.

Proposition 4 *The function $p_0(\alpha_0)$ is increasing in α_0, and $\alpha_0(p_0)$ is increasing in p_0. Moreover, in a symmetric equilibrium with partial market coverage, each provider offers a policy $\wp^{\star}$ such that*

$$p^{\star} = \frac{2}{3}(\Lambda(\alpha^{\star}) - \hat{u}),$$

$$\mu \frac{x^{\star}(1, \alpha^{\star}) - x^{\star}(0, \alpha^{\star})}{2} \sqrt{\frac{\Lambda(\alpha^{\star}) - \hat{u}}{3t}} = c'(\alpha^{\star}).$$

With probability $1 - 2z^{\star}$, L decides not to buy information and invests only based on the prior (i.e., $\hat{x}$).

In the equilibrium with partial market coverage, the level of precision increases with the price, as in the baseline model. However, unlike in the baseline scenario, an equilibrium where the market is not fully covered features a price that increases with precision. The explanation is straightforward:

in this equilibrium, the price must accurately mirror the lender's willingness to purchase information net of their outside option (i.e., the utility of investing based only on priors).[24]

Suppose now that the regulator can control the providers' price and sets it at p. The providers will set accuracy $\alpha(p)$, which increases in p and is determined by the solution of the first-order condition

$$\mu\frac{x^{\star}(1,\alpha)-x^{\star}(0,\alpha)}{2}\frac{p}{2\sqrt{t}\sqrt{\Lambda(\alpha)-\hat{u}-p}}-c'(\alpha)=0. \tag{6}$$

Let

$$z^{\star}(p)\triangleq\sqrt{\frac{\Lambda(\alpha(p))-\hat{u}-p}{t}}.$$

We assume that $z^{\star}(p)\leq\frac{1}{2}$ and verify it later. As before, let $\mathcal{V}(p)$ denote the expected value created by the investment. On each half of the Hotelling line, we then have

$$\frac{\mathcal{V}(p)}{2}\triangleq\int_{0}^{z^{\star}(p)}\left[\Lambda(\alpha(p))-tz^{2}\right]dz+\int_{z^{\star}(p)}^{\frac{1}{2}}\hat{u}dz+\int_{0}^{z^{\star}(p)}\frac{1}{2}\sum_{s=0,1}\phi f(x^{\star}(s,\alpha(p)))\,dz+\int_{z^{\star}(p)}^{\frac{1}{2}}\phi f(\hat{x})\,dz-c(\alpha(p)),$$

We can thus state the following.

Proposition 5 *With partial market coverage and single-homing, the regulator sets a price higher than in the equilibrium with laissez-faire if and only if*

$$\underbrace{\phi\left[\frac{1}{2}\sum_{s=0,1}f(x^{\star}(s,\alpha^{\star}))-f(\hat{x})\right]}_{\textit{Investment efficiency }(+)}\times\underbrace{\frac{dz^{\star}(p^{\star})}{dp}}_{\textit{Marginal type }(?)}+\underbrace{\Lambda'(\alpha^{\star})\,\alpha'(p^{\star})\,z^{\star}(p^{\star})}_{\textit{Inframarginal types }(+)}$$
$$+\underbrace{\frac{\phi\mu}{2(1-\phi)}\left[\frac{1}{\zeta(x^{\star}(1,\alpha^{\star}))}-\frac{1}{\zeta(x^{\star}(0,\alpha^{\star}))}\right]\alpha'(p^{\star})\,z^{\star}(p^{\star})}_{\textit{Baseline effect }(?)}>0.$$

When the market is only partially covered, the regulator faces a more complex trade-off. In addition to internalizing the E's expected profit as in the baseline analysis, the regulator must now also consider how an increased price affects market coverage. Specifically, setting a price above the equilibrium level introduces three new effects. First, it diminishes market coverage, because it makes the option of investing based solely on prior information relatively more attractive to L, thereby reducing investment efficiency as reflected by the positive term

$$\frac{1}{2}\sum_{s=0,1}f(x^{\star}(s,\alpha^{\star}))-f(\hat{x})>0.$$

Second, a higher price enhances precision, which positively affects the marginal lender — i.e., the

[24] In the proof of the proposition we provide sufficient conditions under which the equilibrium exists.

type that is indifferent between acquiring the rating and not. These two effects are reflected by the impact of p on market coverage — i.e., simple algebra shows that

$$\frac{dz^{\star}(p)}{dp} = \frac{\Lambda'(\alpha(p))\alpha'(p) - 1}{2tz^{\star}(p)}, \tag{7}$$

where

$$\Lambda'(\alpha) = \mu \frac{x^{\star}(1,\alpha) - x^{\star}(0,\alpha)}{2} > 0.$$

Hence, the sign of (7) is ambiguous. The first term in the numerator captures the benefit of increased accuracy induced by a higher price, which ceteris paribus expands market coverage and thus improves efficiency. The second term reflects the negative effect of a higher price on coverage, which reduces efficiency by inducing E to rely more frequently on prior beliefs when making investment decisions.

Third, since the market is not fully covered, by increasing precision, a higher price also increases the utility of the inframarginal types — i.e., all lender types (locations) who were buying information before and continue to do so after the price increase.

The net effect is therefore ambiguous and depends on the specific functional form of the revenue function. Even under a CARA specification, the sign of the difference between equilibrium and value-maximizing precision is ambiguous and hinges on the relative strength of the forces described above.

4.2 Equilibrium with multi-homing

In this section, we construct an equilibrium with multi-homing. Given the complexity of the analysis, we focus on the knife-edge case $\phi = 0$, which would imply no regulation under single-homing. This simplification allows us to isolate the key forces that drive optimal regulation in a multi-homing equilibrium. To this purpose, we also assume that L does not invest at the prior — i.e.,

$$f'(0) - 1 < \frac{\mu}{2}, \tag{8}$$

That is, if the two signals have the same precision, L does not invest whenever it receives at least one bad signal (see below for further discussion). The remainder of the game follows the baseline model, with the additional assumption that providers cannot price-discriminate based on the number of ratings the lender purchases (equivalently, on whether it also buys a rating from the rival provider). For expositional simplicity, we further assume linear transport costs, without loss of general insights.

Consider a symmetric equilibrium with the following features:

(i) Both providers offer $\wp^{\star} \triangleq (p^{\star}, \alpha^{\star})$.

(ii) If L is located near the middle of the Hotelling line, then it buys information from both providers.

(iii) If L is located close to P_i, then it only buys information from P_i.

As before, focus on P_0's strategy assuming that P_1 offers the equilibrium policy $\wp^\star$. Before describing the providers' demand functions, it is useful to define L's investment decision when it buys both ratings.[25] Suppose that L has observed signals (s_0, s_1), it solves the following maximization problem

$$\max_{x \geq 0} f(x) - x + \mu x \mathbb{E}\left[\theta | s_0, s_1, \alpha_0, \alpha^\star\right],$$

where $\mathbb{E}\left[\theta | s_0, s_1, \alpha_0, \alpha^\star\right] = \Pr\left[\theta = 1 | s_0, s_1, \alpha_0, \alpha^\star\right]$ and, by Bayes' rule,

$$\Pr\left[\theta = 1 | 1, 1, \alpha_0, \alpha^\star\right] = \frac{\alpha_0 \alpha^\star}{\alpha_0 \alpha^\star + (1-\alpha_0)(1-\alpha^\star)}, \quad \Pr\left[\theta = 1 | 0, 1, \alpha_0, \alpha^\star\right] = \frac{(1-\alpha_0)\,\alpha^\star}{(1-\alpha_0)\,\alpha^\star + \alpha_0\,(1-\alpha^\star)},$$
$$\Pr\left[\theta = 1 | 1, 0, \alpha_0, \alpha^\star\right] = \frac{\alpha_0 (1-\alpha^\star)}{\alpha_0 (1-\alpha^\star) + (1-\alpha_0)\alpha^\star}, \quad \Pr\left[\theta = 1 | 0, 0, \alpha_0, \alpha^\star\right] = \frac{(1-\alpha_0)(1-\alpha^\star)}{(1-\alpha_0)(1-\alpha^\star) + \alpha_0 \alpha^\star}.$$

In an interior solution, L's investment decision is

$$x^\star(s_0, s_1, \alpha_0, \alpha^\star) = \varphi\left(1 - \mu \Pr\left[\theta = 1 | s_0, s_1, \alpha_0, \alpha^\star\right]\right),$$

where $\varphi(\cdot)$ is the inverse of $f'(\cdot)$, as in the baseline model.

Ignoring rating fees and transport costs, the expected benefit to L of acquiring two ratings is

$$\begin{aligned}\Lambda\left(\alpha_0, \alpha^\star\right) \quad &\triangleq \quad \sum_{s_0, s_1} \Pr(s_0, s_1)[f(x^\star(s_0, s_1, \alpha_0, \alpha^\star)) - x^\star(s_0, s_1, \alpha_0, \alpha^\star)] + \\ & \qquad \mu \sum_{s_0, s_1} \Pr(s_0, s_1) \Pr\left[\theta = 1 | s_0, s_1, \alpha_0, \alpha^\star\right] x^\star(s_0, s_1, \alpha_0, \alpha^\star),\end{aligned}$$

and the benefit of acquiring a single rating (say from P_i) is

$$\Lambda\left(\alpha_i\right) \triangleq \frac{1}{2}[f(x^\star(1, \alpha_i)) - x^\star(1, \alpha_i)] + \frac{\mu}{2} \Pr\left[\theta = 1 | 1, \alpha_i\right] x^\star(1, \alpha_i).$$

The following then holds.

Lemma 3 *Abstracting from rating fees and transport costs, L prefers to get more than one informative rating — i.e.,*

$$\Lambda\left(\alpha_0, \alpha^\star\right) \geq \max\left\{\Lambda\left(\alpha_0\right), \Lambda\left(\alpha^\star\right)\right\},$$

Furthermore, in a symmetric equilibrium with $\alpha^\star > \frac{1}{2}$, when L purchases both ratings, investment occurs if and only if it observes two good signals.

The intuition is simple. L benefits from more precise information. Hence, ceteris paribus, he prefers to get more than one informative rating. The fact that, with multi-homing, the investment

[25] If L buys only one rating the investment decision is as in the baseline model.

takes place only in state $(1,1)$ follows directly from the hypothesis that at the prior the lender refrains from financing the entrepreneur.

Equipped with this characterization, we can now turn to define the marginal types. Consider first the location z_- at which L is indifferent between buying only from P_0 and buying from both providers. L's expected utility from multi-homing is

$$u\left(\alpha_0,\alpha^\star,p_0,p^\star,z\right)=\Lambda\left(\alpha_0,\alpha^\star\right)-p_0-p^\star-t,$$

while its utility when buying only from P_0 is

$$u\left(\alpha_0,p_0,z\right)=\Lambda\left(\alpha_0\right)-p_0-tz.$$

The indifferent type is, therefore,

$$z_-\left(\alpha_0,\alpha^\star,p^\star\right)\triangleq 1-\frac{\Lambda\left(\alpha_0,\alpha^\star\right)-\Lambda\left(\alpha_0\right)-p^\star}{t}.$$

Consider now type $z_+(\cdot)$ that is indifferent between purchasing a rating from P_1 only and purchasing ratings from both providers. We have

$$z_+\left(\alpha_0,\alpha^\star,p_0\right)\triangleq\frac{\Lambda\left(\alpha_0,\alpha^\star\right)-\Lambda\left(\alpha^\star\right)-p_0}{t},$$

which is decreasing in p_0, since a higher price for P_0's rating reduces the range of parameters over which L chooses to multi-home. Assuming that

$$0\leq z_-(\cdot)\leq\frac{1}{2}\leq z_+(\cdot)\leq 1,$$

which will be verified later, P_0's maximization problem is

$$\max_{\wp_0\in\mathbb{R}_+\times\left[\frac{1}{2},1\right]} z_+\left(\alpha_0,\alpha^\star,p_0\right)p_0-c\left(\alpha_0\right).$$

Since providers cannot price-discriminate based on the number of ratings that L purchases, the relevant demand margin is determined by the marginal lender farthest from each provider who chooses to multi-home. This margin reflects the interaction between pricing and multi-homing behavior. In a market with competing raters, the decision of the marginal lender — who is indifferent between purchasing one rating or both — shapes each provider's demand. As the price charged by P_0 increases, the likelihood that the lender multi-homes declines, since, in the candidate equilibrium, L is more likely to purchase only the rival's rating.

Assuming the existence of an interior solution, the first-order conditions with respect to p_0 and

α_0 are, respectively,

$$\frac{\partial z^+(\alpha_0,\alpha^\star,p_0)}{\partial p_0}p_0 + z^+(\alpha_0,\alpha^\star,p_0) = 0 \quad \Rightarrow \quad -\frac{1}{t}p_0 + z^+(\alpha_0,\alpha^\star,p_0) = 0,$$

and

$$\frac{\partial z^+(\alpha_0,\alpha^\star,p_0)}{\partial \alpha_0}p_0 - c'(\alpha_0) = 0 \quad \Rightarrow \quad \frac{1}{t}\frac{\partial \Lambda(\alpha_0,\alpha^\star)}{\partial \alpha_0}p_0 = c'(\alpha_0). \tag{9}$$

Strict concavity of the profit function implies that the function $\alpha(p_0)$ solving (9) is increasing in p_0, as in the baseline model. Hence, we can state the following.

Proposition 6 *Suppose that a symmetric equilibrium in which, when located around the center of the Hotelling line, L buys information from both providers, exists. Then, the equilibrium price is*

$$p^\star = \frac{\Lambda(\alpha^\star,\alpha^\star) - \Lambda(\alpha^\star)}{2} > 0,$$

and the equilibrium precision $\alpha^\star$ solves

$$\frac{1}{t}\left[\frac{2\alpha^\star-1}{2}[f(x^\star(1,1,\alpha_0,\alpha^\star)) - x^\star(1,1,\alpha_0,\alpha^\star)] + \frac{\mu\alpha^\star}{2}x^\star(1,1,\alpha_0,\alpha^\star)\right]\frac{\Lambda(\alpha^\star,\alpha^\star) - \Lambda(\alpha^\star)}{2} = c'(\alpha_0)$$

with

$$\Lambda(\alpha^\star,\alpha^\star) = \frac{\alpha^{\star 2} + (1-\alpha^\star)^2}{2}[f(x^\star(1,1,\alpha^\star,\alpha^\star)) - x^\star(1,1,\alpha^\star,\alpha^\star)] + \frac{\mu\alpha^{\star 2}}{2}x^\star(1,1,\alpha^\star,\alpha^\star)$$

and

$$z_+^\star \triangleq 1 - z_-^\star = \frac{\Lambda(\alpha^\star,\alpha^\star) - \Lambda(\alpha^\star)}{2t}.$$

In an equilibrium in which L sometimes purchases two ratings, the price charged by providers reflects the difference between L's utility from acquiring both ratings and its utility from acquiring only one rating.[26]

Consider now value maximization. Following the logic adopted in the baseline model, let $\alpha(p)$ be the function that solves the first-order condition (9) for a given regulated price p, with $\alpha(p^\star) = \alpha^\star$. Suppose that this price is such that L still prefers to multi-home when located around the center of the Hotelling line, while it prefers to single-home when located at the extremes of the segment. Define $\widehat{c}(p) \triangleq c(\alpha(p))$,

$$z_-(p) \triangleq 1 - \frac{\Lambda(\alpha(p),\alpha(p)) - \Lambda(\alpha(p)) - p}{t},$$

[26] In the proof of the proposition we show that such an equilibrium exists if and only if

$$\frac{\Lambda(\alpha^\star,\alpha^\star) - \Lambda(\alpha^\star)}{2} < t < \Lambda(\alpha^\star,\alpha^\star) - \Lambda(\alpha^\star).$$

and

$$z_+ (p) \triangleq \frac{\Lambda(\alpha(p), \alpha(p)) - \Lambda(\alpha(p)) - p}{t},$$

with $\Delta z (p) \triangleq z^+ (p) - z^- (p)$. Furthermore, denote L's expected benefit from one rating by

$$\Lambda_1 (p) \triangleq \frac{1}{2} [f (x^\star (1, \alpha(p))) - x^\star (1, \alpha(p))] + \frac{\mu}{2} \Pr [\theta = 1 | 1, \alpha(p)] \, x^\star (1, \alpha(p)),$$

and from two ratings by

$$\Lambda_2 (p) = \frac{\alpha^2(p) + (1-\alpha(p))^2}{2} [f(x^\star(1,1,\alpha(p),\alpha(p))) - x^\star(1,1,\alpha(p),\alpha(p))] + \frac{\mu\alpha^2(p)}{2} x^\star(1,1,\alpha(p),\alpha(p))$$

Total expected value is

$$\mathcal{V}(p) \triangleq \int_0^{z_-(p)} (\Lambda_1(p) - tz)\, dz + \int_{z_-(\alpha)}^{z_+(\alpha)} (\Lambda_2(p) - t)\, dz + \int_{z_+(\alpha)}^{1} (\Lambda_1(p) - t(1-z))\, dz - 2\widehat{c}(p).$$

Differentiating with respect to p, and evaluating at the laissez-faire equilibrium point $p^\star$ and $\alpha^\star$, we can state the following.

Proposition 7 *Suppose that, when located around the middle of the Hotelling line, L multi-homes. Then, value maximization requires a price floor if and only if*

$$\underbrace{2\left(1 - z_+^\star\right) \mu x^\star (1, \alpha^\star) \, \alpha' (p^\star) + \left(2z_+^\star - 1\right) [2\alpha^\star - 1][f(x^\star(1,1,\alpha^\star,\alpha^\star)) - x^\star(1,1,\alpha^\star,\alpha^\star)]\alpha'^\star)}_{\textit{Inframarginal types}}$$

$$> \underbrace{2\frac{p^\star}{t} \left[\mu x^\star (1, \alpha^\star) \, \alpha' (p^\star) + 2\right] - \mu \alpha^\star x^\star (1,1,\alpha^\star,\alpha^\star) \, \alpha'^\star)}_{\textit{Marginal type}}.$$

This proposition shows that allowing for multi-homing introduces two opposing forces relative to single-homing equilibria. A higher price reduces multi-homing incentives for marginal lenders, as multi-homing becomes more expensive and single-homing more attractive due to higher equilibrium precision. Conversely, higher prices benefit inframarginal lenders by increasing precision while leaving their purchasing decisions unchanged. While the inframarginal effect is always positive, the marginal effect is ambiguous; if negative, a price floor is optimal, whereas if positive, the balance of effects may justify either a price floor or a price cap.

4.3 More than two certification providers

We now consider the Salop (1979)'s circular city version of the baseline model. There are $N \geq 2$ symmetric information providers positioned equidistantly around a circle of circumference 1. We begin with the case where N is exogenous, and discuss the implications of free entry at the end of the section. In both cases, for brevity, we restrict attention to single-homing equilibria and full market coverage.

To simplify exposition, assume again that the transport cost is linear. Hence, L is indifferent between the two nearest providers P_i and P_j if and only if it is located at

$$z = \frac{1}{2N} + \frac{\Lambda(\alpha_i) - p_i - (\Lambda(\alpha_j) - p_j)}{2t},$$

where p_i (resp. p_j) is the rating price charged by P_i (resp. P_j).

Letting p_{i-1} and p_{i+1} be the prices charged by the providers located to the left and to the right of P_i respectively, and assuming interior solutions, the probability that P_i is patronized is

$$q_i(\cdot) = \frac{1}{N} + \frac{p_{i-1} - \Lambda(\alpha_{i-1}) + p_{i+1} - \Lambda(\alpha_{i+1}) + 2(\Lambda(\alpha_i) - p_i)}{2t},$$

Consider an equilibrium in which every provider offers $\wp^\star \triangleq (p^\star, \alpha^\star)$. The above probability simplifies to

$$q_i(\wp_i, \wp^\star) = \frac{1}{N} + \frac{p^\star - \Lambda(\alpha^\star) - (p_i - \Lambda(\alpha_i))}{t}.$$

In a symmetric equilibrium, the first-order conditions associated with P_i's maximization problem yield a price $p^\star(N) = \frac{t}{N}$ and a precision $\alpha^\star(N)$ that solves

$$\frac{1}{N}\frac{\partial \Lambda(\alpha_i)}{\partial \alpha_i} - c'(\alpha_i) = 0. \tag{10}$$

It can be verified that $\alpha^\star(N)$ is decreasing in N — i.e., the larger the number of providers, the lower the probability that each is patronized, thereby reducing the incentive to invest in precision.

Consider now value maximization. Using the same logic as before, we have

$$\mathcal{V}(\alpha) \triangleq \frac{1}{2}\sum_s [f(x^\star(s,\alpha)) - x^\star(s,\alpha)] + \frac{\mu}{2}\sum_s \Pr[s|\theta = 1]\, x^\star(s,\alpha) - \frac{t}{4N} - Nc(\alpha).$$

The derivative of this function leads to the same prediction as in the baseline model: providers either under-invest or over-invest in precision, depending on whether the concavity index of the revenue function is decreasing or increasing. Therefore, with an exogenous number of providers, the policy implications of the baseline model remain valid.

Endogenous entry. Suppose that providers must pay a fixed set-up cost $F > 0$ to enter the market. The timing of the game changes only with respect to the entry stage. At the outset of the game, providers enter the information market and locate equidistantly along the Salop circle.[27] The game then unfolds as before.

While the first-order conditions with respect to prices and precision levels remain unchanged

[27] As standard, we treat N as a continuous variable.

with respect to the above analysis, the number $N^\star$ of providers that enter the market when there is no regulatory oversight is determined by the following zero-profit condition

$$\frac{t}{N^2} - c\left(\alpha^\star\left(N\right)\right) - F = 0,$$

with $N^\star \geq 2$ for F sufficiently small. Let $\alpha^\star\left(N^\star\right) = \alpha^\star$ and $p^\star\left(N^\star\right) = p^\star$ be the equilibrium values of precision and price, respectively.

Focus now on value maximization. Let p be the price mandated by the regulator. Given this price, the two conditions that identify the number of active providers $N^\star\left(p\right)$ and the level of precision $\alpha^\star\left(p\right)$ that they supply, are the free-entry condition and the first-order condition with respect to precision — i.e.,

$$\frac{p}{N} - c\left(\alpha\right) - F = 0, \tag{11}$$

$$\frac{1}{t}\frac{\partial\Lambda\left(\alpha\right)}{\partial\alpha_i}p - c'\left(\alpha\right) = 0. \tag{12}$$

Notice that (12) does not depend on N and the second-order condition implies that $\alpha^\star\left(p\right)$ is increasing in p. The first condition (free-entry) then implies

$$N\left(p\right) \triangleq \frac{p}{c(\alpha(p)) + F} \quad\Rightarrow\quad N'\left(p\right) = \frac{c(\alpha(p)) + F - pc'(\alpha(p))\alpha'(p)}{[c(\alpha(p)) + F]^2}.$$

This means that an increase in the rating price has an ambiguous effect on entry. On one hand, a higher price enhances profit margins, which can stimulate entry by making the market more attractive to new providers. On the other hand, a higher price leads competing providers to supply greater precision, which either reduces a given provider's market share or forces this provider to supply greater precision, which is costly. The increased certification cost can deter new entrants from joining the market. The net effect of a price increase on entry depends on the balance between these two opposing forces: the allure of higher potential profits and the discouragement of a higher cost of providing a rating.

Hence, as in the Hotelling model with a partially covered market, endogenous entry introduces additional dynamics that — beyond the role of the curvature of the revenue function — depend on how an increase in price affects transport costs, fixed costs, and certification costs. At a given p, expected total value is

$$\begin{aligned}\mathcal{V}\left(\alpha\left(p\right), N\left(p\right)\right) \quad &\triangleq \quad \frac{1}{2}\sum_s \left[f\left(x^\star\left(s, \alpha\left(p\right)\right)\right) - x^\star\left(s, \alpha\left(p\right)\right)\right] + \\ &\quad \frac{\mu}{2}\sum_s \Pr\left[s|\theta = 1, \alpha\left(p\right)\right] x^\star\left(s, \alpha\left(p\right)\right) - \frac{t}{4N\left(p\right)} - N\left(p\right)\left(c\left(\alpha\left(p\right)\right) + F\right).\end{aligned}$$

An increase (resp. reduction) in the number of providers induced by an increase in the price has two direct effects on social costs: it reduces (resp. increases) the lender's transport cost, but at the same time it also increases (resp. reduces) certification and fixed entry costs. In an equilibrium with free entry, providers do not internalize this effect, leading to the classical excessive entry result (see, e.g., Polo, 2018, for a discussion of entry models and excess entry).

We can state the following.

Proposition 8 *The derivative of the project's value with respect to p evaluated at the equilibrium price $p^\star$ and precision $\alpha^\star$ is*

$$\frac{d\mathcal{V}(\alpha^\star, N^\star)}{dp} = \frac{\phi\mu}{2(1-\phi)}\left[\frac{1}{\zeta(x^\star(1,\alpha^\star))} - \frac{1}{\zeta(x^\star(0,\alpha^\star))}\right]\alpha'(p^\star) - \frac{3t}{4N^{\star 2}}N'(p^\star).$$

Hence, compared to the case with exogenous entry, in an equilibrium with free-entry, a price floor is relatively more likely to be socially optimal if $N(p^\star)$ is decreasing in $p^\star$, otherwise a price-cap is relatively more likely to maximize value.

With endogenous entry, the justification for a price floor becomes more compelling when the dominant effect on entry is the increase in certification costs. Essentially, a rise in price tends to increase certification costs, which leads to fewer providers entering the market. This effect mitigates the traditional excessive entry problem. Conversely, in cases where the prevailing effect on entry is an increase in profit margins, a rise in price encourages more entrants, exacerbating the excessive entry dynamics, making a price cap relatively more desirable.

4.4 The issuer-pay model

To contrast our results with a traditional rating agency framework, we now consider the opposite scenario of an issuer-pay model. With this business model, E pays the providers for the information that L uses to make its investment decisions. The rest of the game remains as in the baseline model with three important caveats. First, we assume that the lender can observe the information policies offered by the two raters. Second, to avoid signaling issues that will be discussed later, we assume that E does not know the project type when it requests a rating, and the rating becomes public when issued. Third, E instead of L is now located on the Hotelling line between the providers. As in the baseline model, we focus on an equilibrium with full market coverage and single-homing.

The optimal loan size in a symmetric equilibrium with a covered market and single-homing remains $x^\star(s_i, \alpha_i)$. Therefore, the profit of an entrepreneur with a type-θ project is

$$\begin{aligned}\pi_i(\alpha_i, p_i, \theta) &\triangleq \phi\sum_{s_i}\Pr[s_i|\theta,\alpha_i]\, f(x^\star(s_i,\alpha_i)) - p_i - td_i \\ &= \begin{cases}\phi[\alpha_i f(x^\star(1,\alpha_i)) + (1-\alpha_i) f(x^\star(0,\alpha_i))] - p_i - td_i & \text{if } \theta = 1 \\ \\ \phi[(1-\alpha_i) f(x^\star(1,\alpha_i)) + \alpha_i f(x^\star(0,\alpha_i))] - p_i - td_i & \text{if } \theta = 0\end{cases}.\end{aligned}$$

Hence, its expected utility is

$$\pi_i(\alpha_i, p_i) \triangleq \sum_\theta \Pr[\theta]\, \pi_i(\alpha_i, p_i, \theta) = \frac{\phi}{2} \sum_{s_i} f(x^\star(s_i, \alpha_i)) - p_i - td_i$$

The indifferent entrepreneur is located at

$$z(\wp_0, \wp_1) \triangleq \frac{1}{2} + \frac{p_1 - p_0}{2t} + \phi \frac{\sum_{s_0} f(x^\star(s_0, \alpha_0)) - \sum_{s_1} f(x^\star(s_1, \alpha_1))}{4t}$$

which, again, equals the probability with which E patronizes P_0.

Focusing again on a symmetric equilibrium in which both providers offer policy $\wp^* \triangleq (\alpha^*, p^*)$, provider P_0 solves the following problem:

$$\max_{\wp_0} z(\wp_0, \wp^*)\, p_0 - c(\alpha_0).$$

When the market is covered, the first-order condition with respect to p_0 yields again the standard Hotelling pricing rule $p^* = t$. The first-order condition with respect to α_0 is again

$$\frac{\partial z(\wp_0, \wp^*)}{\partial \alpha_0} p_0 - c'(\alpha_0) = 0, \tag{13}$$

where now
$$\frac{\partial z(\wp_0, \wp^*)}{\partial \alpha_0} = \frac{\phi\mu}{4t(1-\phi)} \left[\frac{1}{\zeta(x^\star(1, \alpha_0))} - \frac{1}{\zeta(x^\star(0, \alpha_0))} \right],$$
which is positive if $\zeta'(x) < 0$ for every $x \geq 0$, and negative if $\zeta'(\cdot) > 0$, because $x^\star(1, \alpha) > x^\star(0, \alpha)$.

The following then holds:

Proposition 9 *Suppose that E is uninformed when it chooses the information provider (or, alternatively, that it is informed but the game features a pooling equilibrium) and that both providers operate under an issuer-pay business model. Then, under laissez-faire, a symmetric equilibrium with full coverage and single-homing features a price $p^* = t$ and a level of information precision $\alpha^* \in (0, 1)$ if $\zeta'(x) < 0$ for every $x \geq 0$. In this case, α^* solves*

$$\frac{\phi\mu}{4(1-\phi)} \left(\frac{1}{\zeta(x^\star(1, \alpha))} - \frac{1}{\zeta(x^\star(0, \alpha))} \right) = c'(\alpha^*),$$

and is increasing in μ and ϕ. By contrast, if $\zeta'(x) \geq 0$ for every $x \geq 0$, then both providers supply uninformative signals, and the lender makes an uninformed choice, which may be to not finance the project (market collapse).

Value maximization mandates a higher level of precision than the level of precision that emerges with laissez-faire. Hence, from a regulatory point of view, a price floor that induces providers to price above t is always optimal.

Under the issuer-pay model, providers are reluctant to supply informative ESG ratings when the revenue function features IARA. In this case, ratings become uninformative because, as explained above, providers internalize the fact that increased precision benefits the revenues of a type $\theta = 1$ project less than it reduces the revenues of a type $\theta = 0$ project. As a result, E's expected utility, and the demand for ratings, decreases with precision, leading to rating degradation and uninformed investment decisions by L, or even to market collapse (i.e., the project is not financed). Conversely, when the marginal revenue is DARA, providers sell informative signals, but this level of precision is inefficiently low from a value maximization point of view, because profit maximization under the issuer-pay model does not consider L's expected utility, which benefits from greater precision. Thus, under an issuer-pay model, the optimal regulation invariably requires a price floor, and never a price cap.

Remark. In the above analysis, we assumed that E is uninformed about the project type at the stage in which it requests a rating. However, how would the results change if E learns θ before requesting a rating? In this case, the nature of the game becomes more complex: the equilibrium concept must account for the fact that E has private information. Hence, its rating acquisition decisions might signal this information. Since the game is sequential, the effectiveness of this information transmission depends on how L's beliefs are formed off the equilibrium path. For this reason, the appropriate equilibrium concept is weak Perfect Bayesian Equilibrium (weak PBE). As in any signaling game, the equilibrium set can be quite large — there can be separating, pooling, and semi-separating equilibria. Pooling equilibria are easy to characterize and are sustained by off-equilibrium beliefs such that given a candidate equilibrium $\wp^{\star}$, when a provider deviates from this policy and E buys a rating from the deviator, L assigns the same probability to both project types before seeing the rating. With such an off-path belief, the equilibrium characterization remains as above, since E's action of accepting an off-equilibrium policy does not signal anything to L.

By contrast, separating equilibria are less straightforward to characterize. This is because any such equilibrium must necessarily be asymmetric in the sense that raters must specialize in rating different projects — e.g., P_0 rates only type-0 projects and P_1 rates only type-1 projects. However, in addition to the standard incentive compatibility constraint for the entrepreneur — i.e., an entrepreneur with a type 0 project must not want to mimic the behavior of an entrepreneur with a type 1 project, and vice-versa — raters must not have an incentive to deviate to offering a policy that attracts all project types (a logic similar to the seminal work by Rothschild and Stiglitz, 1978). In addition, in these equilibria where information unravels, there is no need for L to gather information, which makes the entrepreneur's mimicking incentive even stronger. Examining these additional aspects deserves attention on its own. Yet, since the focus of this paper is the subscriber-pay model, we leave such an analysis for future research.

5 Concluding remarks

ESG ratings are crucial for guiding investments towards sustainable practices. Yet, they face challenges, such as potential conflicts of interest and transparency issues, similar to traditional credit ratings. Regulatory interventions can sometimes mitigate these problems, particularly concerning the relationship between raters' revenues and the entities they evaluate. Despite the fact that the majority of ESG raters adopt a subscriber-pay model, which should reduce the potential for conflicts of interest compared to an issuer-pay model, the question remains as to whether this is sufficient to ensure the effectiveness of ESG ratings, or if additional regulatory measures are necessary to support truly sustainable investment decisions. Our analysis provides a first answer to this question. We show that regulatory measures, such as price floors and price caps, can be useful in ensuring the effectiveness of ESG ratings, but only if they are well designed. These measures address market inefficiencies where information precision may be under-supplied or over-supplied relative to what would maximize value. In particular, we find that when the market is under-supplying rating information precision, a price floor should be considered, and when the market over-supplies rating precision, a price cap should instead be considered. Our findings indicate that indiscriminate imposition of price controls, such as universal price caps or floors, can significantly reduce overall value and welfare. This emphasizes the need for nuanced regulatory strategies tailored to specific market conditions.

References

[1] Azarmsa E., and J., Shapiro (2024), "The Market for ESG Ratings", mimeo.

[2] Admati, A., and P. Pfleiderer (1986), "A Monopolistic Market for Information." *Journal of Economic Theory*, 39(2), 400-438.

[3] Admati, A., and P. Pfleiderer (1990), "Direct and Indirect Sale of Information." *Econometrica*, 58(4), 901-928.

[4] Bar-Isaac, H. and J. Shapiro (2013). "Ratings quality over the business cycle". *Journal of Financial Economics* 108(1), 62–78.

[5] Berg, F., J. F. Koelbel, and R. Rigobon (2022). "Aggregate confusion: The divergence of ESG ratings." *Review of Finance.*

[6] Bolton, P., Freixas, X., & Shapiro, J. (2012). "The credit ratings game." The Journal of Finance, 67(1), 85-111.

[7] Chatterji, A. K., R. Durand, D. I. Levine, and S. Touboul (2016). "Do ratings of firms converge? Implications for managers, investors and strategy researchers." *Strategic Management Journal* 37(8), 1597–1614.

[8] Balestrieri, F., and S. Izmalkov (2014), "Informed Seller in a Hotelling Market." Mimeo, New Economic School.

[9] Bergemann, D. and A. Bonatti (2019). "Markets for information: An introduction." *Annual Review of Economics* 11, 85–107.

[10] Bergemann, D., A. Bonatti, and A. Smolin (2018). "The design and price of information." *American Economic Review,* 108(1), 1–48

[11] Celik, L. (2014), "Information Unraveling Revisited: Disclosure of Horizontal Attributes." *Journal of Industrial Economics*, 62(1), 113-136.

[12] Gollier, C. (2001). The economics of risk and time. MIT press.

[13] Hörner, J., and A. Skrzypacz (2016), "Selling Information." *Journal of Political Economy*, 124, 1515-1562.

[14] Huang, S., L. Yang, and Y. Xiong (2018). "Clientele, information sales, and asset prices." *Information Sales, and Asset Prices* (January 30, 2018).

[15] Kamenica, E., and M. Gentzkow (2011), "Bayesian Persuasion." *American Economic Review,* 101(6), 2590-2615.

[16] Kastl, J., Pagnozzi, M., and Piccolo, S. (2018). "Selling information to competitive firms." *The RAND Journal of Economics*, 49(1), 254-282.

[17] Koessler, F., and V. Skreta (2014), "Sales Talk." Mimeo, PSE and UCL.

[18] Li, H., and X. Shi (2013), "Discriminatory Information Disclosure." Mimeo, University of British Columbia and University of Toronto.

[19] Lizzeri, A. (1999), "Information Revelation and Certification Intermediaries." *RAND Journal of Economics*, 30(2), 214-231.

[20] Mylovanov, T., and T. Tröger (2014), "Mechanism Design by an Informed Principal: the Quasi-Linear Private-Values Case." *Review of Economic Studies*, 81, 1668-1707.

[21] Piccolo, A. (2021). Credit ratings and competition. Available at SSRN 3233964

[22] Piccolo, S., P. Tedeschi and G. Ursino (2015), "How Limiting Deceptive Practices Harms Consumers." *RAND Journal of Economics*, 46, 611-624.

[23] Polo, M. (2018). Entry games and free entry equilibria. (pp. 312-342). Edward Elgar Publishing.

[24] Rayo, L., and I. Segal (2010), "Optimal Information Disclosure." *Journal of Political Economy*, 118(5), 949-987.

[25] Rothschild, M., and Stiglitz, J. (1978). Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. In Uncertainty in economics (pp. 257-280). Academic Press.

Appendix

Proof of Lemma 1. Conditional on signal s_i and precision $\alpha_i \geq \frac{1}{2}$, the lender's first order condition is

$$(1-\phi) f'(x) - 1 + \mu \mathbb{E}[\theta | s_i, \alpha_i] = 0 \quad \Leftrightarrow \quad x^\star(s_i, \alpha_i) \triangleq \varphi\left(\frac{1-\mu \mathbb{E}[\theta | s_i, \alpha_i]}{1-\phi}\right),$$

with

$$\mathbb{E}[\theta | s_i, \alpha_i] = \begin{cases} \alpha_i & \text{if } s_i = 1, \\ 1-\alpha_i & \text{if } s_i = 0. \end{cases}$$

Hence, $f''(\cdot) < 0$ and $\alpha_i \geq \frac{1}{2}$ immediately imply $x^\star(1, \alpha_i) \geq x^\star(0, \alpha_i) > 0$. Furthermore,

$$\frac{\partial x^\star(1, \alpha_i)}{\partial \alpha_i} = -\frac{\mu}{1-\phi} \frac{1}{f''(x^\star(1, \alpha_i))} > 0 > \frac{\partial x^\star(0, \alpha_i)}{\partial \alpha_i} = \frac{\mu}{1-\phi} \frac{1}{f''(x^\star(0, \alpha_i))}. \quad \blacksquare$$

Proof of Lemma 2. From the first-order condition (2), and $p^* = t$, $\frac{\partial z(\cdot)}{\partial p} = -\frac{1}{2t}$, we have

$$\begin{aligned} \frac{\partial \hat{\alpha}_0(p_0)}{\partial p_0} &= -\frac{\mu \frac{x^\star(1,\alpha_0) - x^\star(0,\alpha_0)}{4t}}{\mu \frac{\mu}{1-\phi} \frac{-\frac{1}{f''(x^\star(1,\alpha_0))} - \frac{1}{f''(x^\star(0,\alpha_0))}}{4t} p_0 - c''(\alpha_0)} \\ &= -\frac{x^\star(1,\alpha_0) - x^\star(0,\alpha_0)}{-\frac{\mu}{1-\phi}\left(\frac{1}{f''(x^\star(1,\alpha_0))} + \frac{1}{f''(x^\star(0,\alpha_0))}\right) t - \frac{4t}{\mu} c''(\alpha_0)}, \end{aligned}$$

which is positive, because we assumed that $c''(\cdot)$ is sufficiently large and $f(\cdot)$ has a bounded third-order derivative. Finally, $\lim_{p_0 \to 0} \hat{\alpha}_0(p_0) = \frac{1}{2}$ follows directly from (2) and the bounded $\frac{\partial z(\cdot)}{\partial \alpha_0}$. $\blacksquare$

Proof of Proposition 1. To begin with, notice that $p^\star = t$ is an immediate solution of condition (1) when the providers offer symmetric policies. The condition that pins down $\alpha^\star$, instead, follows from (2), whose solution lies in the interval $\left[\frac{1}{2}, 1\right]$ from the Inada conditions and the fact that $x^\star(1, \alpha^\star) > x^\star(0, \alpha^\star)$. Strict concavity of the provider's profit function in α_0 then implies the comparative statics with respect to μ and ϕ.

Finally, to show that the market is covered and the lender single-homes, note that L's expected utility when located at $\frac{1}{2}$ is

$$\frac{1}{2} \sum_{s_i} \left[(1-\phi) f(x^\star(s_i, \alpha^\star)) - x^\star(s_i, \alpha^\star) + \frac{\mu}{2} \Pr[\theta = 1 | s_i, \alpha^\star] x^\star(s_i, \alpha^\star)\right] - \frac{t}{4} - t = \Lambda(\alpha^\star) - \frac{5}{4} t.$$

Taking the difference with $\hat{u}$, the lender prefers to purchase information if and only if

$$\Lambda(\alpha^\star) - \hat{u} > \frac{5t}{4}. \tag{A1}$$

Suppose now that the lender deviates from the equilibrium with single-homing and purchases both signals. Assuming that L has observed signals (s_0, s_1), it solves the following maximization problem

$$\max_{x\geq 0} (1-\phi) f(x) - x + \mu x \mathbb{E}[\theta|s_0, s_1, \alpha^\star, \alpha^\star].$$

By Bayes' rule, we have

$$\Pr[\theta = 1|1,1,\alpha^\star,\alpha^\star] = \frac{\alpha^{\star 2}}{\alpha^{\star 2} + (1-\alpha^\star)^2}, \quad \Pr[\theta = 1|0,1,\alpha^\star,\alpha^\star] = \frac{1}{2},$$
$$\Pr[\theta = 1|1,0,\alpha^\star,\alpha^\star] = \frac{1}{2}, \quad \Pr[\theta = 1|0,0,\alpha^\star,\alpha^\star] = \frac{(1-\alpha^\star)^2}{(1-\alpha^\star)^2 + \alpha^{\star 2}}.$$

In an interior solution, L's investment decision is

$$x^\star(s_0, s_1, \alpha^\star, \alpha^\star) = \varphi\left(\frac{1 - \mu \Pr[\theta = 1|s_0, s_1, \alpha^\star, \alpha^\star]}{1-\phi}\right),$$

where $\varphi(\cdot)$ is the inverse of $f'(\cdot)$. Notice that

$$x^\star(0,1,\alpha^\star,\alpha^\star) = x^\star(1,0,\alpha^\star,\alpha^\star) = \hat{x}.$$

Without accounting for the service price and the transport cost, the lender's expected benefit of acquiring two ratings is

$$\Lambda(\alpha^\star,\alpha^\star) \triangleq \frac{\alpha^{\star 2} + (1-\alpha^\star)^2}{2}[f(x^\star(1,1,\alpha^\star,\alpha^\star)) - x^\star(1,1,\alpha^\star,\alpha^\star)] +$$
$$\frac{\alpha^{\star 2}\mu}{2} \Pr[\theta = 1|1,1,\alpha^\star,\alpha^\star]\, x^\star(1,1,\alpha^\star,\alpha^\star) + 2\alpha^\star(1-\alpha^\star)\hat{u} +$$
$$\frac{\alpha^{\star 2} + (1-\alpha^\star)^2}{2}[f(x^\star(0,0,\alpha^\star,\alpha^\star)) - x^\star(0,0,\alpha^\star,\alpha^\star)] + \frac{(1-\alpha^\star)^2\mu}{2} \Pr[\theta = 1|0,0,\alpha^\star,\alpha^\star]\, x^\star(0,0,\alpha^\star,\alpha^\star),$$

with $\Lambda(\alpha^\star,\alpha^\star) > \Lambda(\alpha^\star)$ because, other things being equal, acquiring more information is always better for the lender.

Hence, the lender prefers to single home if and only if

$$\Lambda(\alpha^\star) - \frac{t}{4} > \Lambda(\alpha^\star,\alpha^\star) - t \quad \Leftrightarrow \quad \frac{3t}{4} > \Lambda(\alpha^\star,\alpha^\star) - \Lambda(\alpha^\star). \tag{A2}$$

Summing up, conditions (A2) and (A1) yield

$$\frac{4(\Lambda(\alpha^\star) - \hat{u})}{5} > t > \frac{4(\Lambda(\alpha^\star,\alpha^\star) - \Lambda(\alpha^\star))}{3}. \tag{A3}$$

Notice that $\alpha^\star$, $\Lambda(\alpha^\star)$ and $\Lambda(\alpha^\star,\alpha^\star)$ do not depend on t. Furthermore, at $\alpha^\star = 1$, the condition (A3) never defines an empty set, because $\Lambda(1) > \hat{u}$ and $\Lambda(\alpha^\star,\alpha^\star) = \Lambda(\alpha^\star)$. Hence, by continuity, it must hold for $\alpha^\star$ close to 1, which occurs if $c'(\cdot)$ is small enough. The rest of the proof follows immediately. ■

Concavity of the project's value. To show concavity of the value function, notice that in a symmetric equilibrium,

$$\mathcal{V}''(\alpha) = -\frac{\mu^2}{2(1-\phi)}\left[\frac{1}{f''(x^\star(1,\alpha))} + \frac{1}{f''(x^\star(0,\alpha))}\right] - 2c''(\alpha) + \frac{\phi\mu^2}{2(1-\phi)^2}\left[\frac{[f''(x^\star(1,\alpha))]^2 - f'(x^\star(1,\alpha))f'''(x^\star(1,\alpha))}{[f''(x^\star(1,\alpha))]^3} + \frac{[f''(x^\star(0,\alpha))]^2 - f'(x^\star(0,\alpha))f'''(x^\star(0,\alpha))}{[f''(x^\star(0,\alpha))]^3}\right].$$

If $f'(\cdot)$, $f''(\cdot)$ and $f'''(\cdot)$ are bounded away from zero and infinity, and $c''(\cdot)$ is large enough, then $\mathcal{V}''(\alpha) < 0$.

Proof of Proposition 2. Evaluating (3) at the laissez-faire $\alpha^\star$ we have

$$\mathcal{V}'(\alpha^\star) = \frac{\phi\mu}{2(1-\phi)}\left[\frac{1}{\zeta(x^\star(1,\alpha^\star))} - \frac{1}{\zeta(x^\star(0,\alpha^\star))}\right],$$

which, for $\phi > 0$, yields immediately the result since $x^\star(0,\alpha^\star) < x^\star(1,\alpha^\star)$. The result is trivial at $\phi = 0$. ■

Proof of Proposition 3. The first part of the proof follows immediately from the fact that the function $\hat{\alpha}(p)$ is increasing and from the results stated in Proposition 2. The second part of the proof follows immediately by comparing $p^{\star\star}$ and $p^\star$ and noticing that this difference is proportional to t. ■

Proof of Proposition 4. The fact that $p_0(\alpha_0)$ is increasing in α_0 follows immediately from

$$\Lambda'(\alpha_0(p)) = \mu\frac{x^\star(1,\alpha_0(p)) - x^\star(0,\alpha_0(p))}{2} > 0.$$

The fact that $\alpha_0(p_0)$ is increasing in p_0 follows from the assumption that $c(\cdot)$ is sufficiently convex, so P_0's objective function is concave in α_0 (the SOC holds), and the fact that the term

$$p_0\frac{1}{\sqrt{t}\sqrt{\Lambda(\alpha_0) - \hat{u} - p_0}}$$

is increasing in p_0.

The equilibrium conditions are derived by substituting the equilibrium price into the first-order condition with respect to α_0 evaluated at a symmetric equilibrium. To ensure that the lender equidistant between the providers does not buy a rating, note that $\Lambda(\alpha^\star) > \hat{u}$ and by definition

$$z^\star \triangleq \sqrt{\frac{\Lambda(\alpha^\star) - \hat{u}}{3t}}.$$

If $\alpha^\star$ is close to $\frac{1}{2}$, then $z^\star$ is close to 0, because $\Lambda\left(\frac{1}{2}\right) = \hat{u}$ and $\Lambda(\cdot)$ is continuous. Intuitively, almost nobody buys ratings that are nearly uninformative. Of course, $\alpha^\star$ is endogenous, but if $c'(\cdot)$ is large enough (rating precision costly enough), then $\alpha^\star$ is close to $\frac{1}{2}$. In this case, the lender never wants to multi-home irrespective of its location — i.e.,

$$\Lambda\left(\alpha^{\star},\alpha^{\star}\right)-2p^{\star}-t<\Lambda\left(\alpha^{\star}\right)-p^{\star}-z^{\star 2}t=\hat{u}\quad\Rightarrow\quad\Lambda\left(\alpha^{\star},\alpha^{\star}\right)-\hat{u}-\frac{4}{3}\left(\Lambda\left(\alpha^{\star}\right)-\hat{u}\right)<t,$$

which is always satisfied at $\alpha^{\star}=\frac{1}{2}$, since $\Lambda\left(\alpha^{\star},\alpha^{\star}\right)=\Lambda\left(\alpha^{\star}\right)=\hat{u}$, and hence by continuity it holds for $\alpha^{\star}$ sufficiently close to $\frac{1}{2}$. ■

Proof of Proposition 5. On each half of the Hotelling line, we have

$$\frac{\mathcal{V}\left(p\right)}{2}\triangleq\int_{0}^{z^{\star}(p)}\left[\Lambda\left(\alpha\left(p\right)\right)-tz^{2}\right]dz+\int_{z^{\star}(p)}^{\frac{1}{2}}\hat{u}dz-c\left(\alpha\left(p\right)\right)+\int_{0}^{z^{\star}(p)}\frac{1}{2}\sum_{s}\phi f\left(x^{\star}\left(s,\alpha\left(p\right)\right)\right)dz+\int_{z^{\star}(p)}^{\frac{1}{2}}\phi f\left(\hat{x}\right)dz,$$

which simplifies to

$$\frac{\mathcal{V}\left(p\right)}{2}=z^{\star}\left(p\right)\Lambda\left(\alpha\left(p\right)\right)-\frac{t(z^{\star}\left(p\right))^{3}}{3}+\left[\frac{1}{2}-z^{\star}\left(p\right)\right]\hat{u}-c\left(\alpha\left(p\right)\right)+z^{\star}\left(p\right)\frac{\phi}{2}\sum_{s}f\left(x^{\star}\left(s,\alpha\left(p\right)\right)\right)+\left[\frac{1}{2}-z^{\star}\left(p\right)\right]\phi f\left(\hat{x}\right).$$

Differentiating and using (6),

$$\begin{aligned}\frac{\mathcal{V}'\left(p\right)}{2}=&\left[p-\phi f\left(\hat{x}\right)+\frac{\phi}{2}\sum_{s}f\left(x^{\star}\left(s,\alpha\left(p\right)\right)\right)\right]\frac{dz^{\star}\left(p\right)}{dp}+\Lambda'\left(\alpha\left(p\right)\right)\alpha'\left(p\right)z^{\star}\left(p\right)\\&+\frac{\phi\mu}{2(1-\phi)}\left[\frac{1}{\zeta(x^{\star}\left(1,\alpha\left(p\right)\right))}-\frac{1}{\zeta(x^{\star}\left(0,\alpha\left(p\right)\right))}\right]\alpha'\left(p\right)z^{\star}\left(p\right)-c'\left(\alpha\left(p\right)\right)\alpha'\left(p\right).\end{aligned}$$

Evaluating $\mathcal{V}'\left(p\right)$ at $p^{\star}$, the result follows immediately. ■

Proof of Lemma 3. The inequality

$$\Lambda\left(\alpha_{0},\alpha^{\star}\right)\geq\max\left\{\Lambda\left(\alpha_{0}\right),\Lambda\left(\alpha^{\star}\right)\right\},$$

follows from a straightforward revealed preference argument: if the additional information that the lender buys is useless, then the lender can always ignore it. However, for $\alpha_0>\frac{1}{2}$ and $\alpha^{\star}>\frac{1}{2}$, the lender will never ignore a rating. In fact, it is easy to show that $x^{\star}(s_0,s_1,\alpha_0=\frac{1}{2},\alpha^{\star})=x^{\star}(s_1,\alpha^{\star})$ for every $\alpha^{\star}\geq\frac{1}{2}$ (and vice-versa). If one of $\alpha_0,\alpha^{\star}$ equals 1, then the lender already gets perfect information from that, so the other rating adds nothing.

The fact that $x^{\star}(s_0,s_1,\alpha^{\star},\alpha^{\star})>0$ if and only if $s_0=s_1=1$ is a direct consequence of assumption (8): if the lender does not invest at the prior, a fortiori it will not invest when receiving two bad signals. ■

Proof of Proposition 6. The conditions that identify the symmetric equilibrium $p^{\star}$ and $\alpha^{\star}$ are immediate to obtain from the providers' first-order conditions. Existence, instead, requires first that

$$z_{+}^{\star}=\frac{\Lambda\left(\alpha^{\star},\alpha^{\star}\right)-\Lambda\left(\alpha^{\star}\right)}{2t}\in\left(\frac{1}{2},1\right)$$

that is,

$$\Lambda(\alpha^\star,\alpha^\star)-\Lambda(\alpha^\star)>t,$$

which also implies that $z_+^\star > z_-^\star$, and

$$\frac{\Lambda(\alpha^\star,\alpha^\star)-\Lambda(\alpha^\star)}{2}<t.$$

Furthermore, the lender must not want not deviate. There are two types of deviations: (*i*) a lender that is supposed to purchase two ratings in equilibrium decides to buy only one; (*ii*) a lender that is supposed to buy only one rating in equilibrium decides to buy both.

Consider the first type of deviation. Focus, without loss of generality, on the first half of the Hotelling segment. Notice that, by construction

$$\Lambda(\alpha^\star,\alpha^\star)-2p^\star-t=\Lambda(\alpha^\star)-p^\star-z_-^\star t \tag{A4}$$

Hence, for every $z<z_-^\star$, we have

$$\Lambda(\alpha^\star,\alpha^\star)-2p^\star-t<\Lambda(\alpha^\star)-p^\star-z_-^\star t.$$

By symmetry, the same applies to the second half of the Hotelling segment. Mutatis-mutandis, equality (A4) also implies that the second type of deviation is not profitable. Hence, existence simply requires

$$\frac{\Lambda(\alpha^\star,\alpha^\star)-\Lambda(\alpha^\star)}{2}<t<\Lambda(\alpha^\star,\alpha^\star)-\Lambda(\alpha^\star),$$

which always defines a non-empty set for every $\alpha^\star\in\left(\frac{1}{2},1\right)$. ■

Proof of Proposition 7. Assuming that the price dictated by the regulator does not alter the characteristics of the equilibrium — i.e., that sellers located in the middle of the Hotelling line multi-home while those located at the periphery single-home, the total expected value is

$$\mathcal{V}(p)\triangleq\int_0^{z_-(p)}(\Lambda_1(p)-tz)\,dz+(\Lambda_2(p)-t)\int_{z_-(p)}^{z_+(p)}dz+\int_{z_+(p)}^{1}(\Lambda_1(p)-t(1-z))\,dz-2\widehat{c}(p).$$

Differentiating with respect to p yields

$$\begin{aligned}\mathcal{V}'(p) &= (\Lambda_1(p)-tz_-(p))\frac{dz_-(p)}{dp}+z_-(p)\frac{d\Lambda_1(p)}{dp}+\left(\frac{dz_+(p)}{dp}-\frac{dz_-(p)}{dp}\right)(\Lambda_2(p)-t)+\\ &\quad (z_+(p)-z_-(p))\frac{d\Lambda^2(p)}{dp}-(\Lambda_1(p)-t(1-z_+(p)))\frac{dz_+(p)}{dp}+(1-z_+(p))\frac{d\Lambda_1(p)}{dp}-2\,\widehat{c}'(p).\end{aligned}$$

Using the fact that

$$z_+^\star(p)=1-z_-^\star(p)=\frac{\Lambda(\alpha(p),\alpha(p))-\Lambda(\alpha(p))-p}{t},$$

the above simplifies to

$$\mathcal{V}'(p) = -2\left[\Lambda_1(p) - t(1-z_+(p)) - (\Lambda_2(p) - t)\right]\frac{dz_+(p)}{dp} + 2(1-z_+(p))\frac{d\Lambda_1(p)}{dp} + (2z_+(p)-1)\frac{d\Lambda_2(p)}{dp} - 2c'(\alpha)\alpha'(p).$$

where, by the Envelope Theorem,

$$\frac{d\Lambda_1(p)}{dp} = \frac{\mu}{2}x^\star(1,\alpha(p))\alpha'(p) > 0,$$

$$\frac{d\Lambda_2(p)}{dp} = \frac{2\alpha(p)-1}{2}[f(x^\star(1,1,\alpha(p),\alpha(p))) - x^\star(1,1,\alpha(p),\alpha(p))]\alpha'(p) + \frac{\mu\alpha(p)}{2}x^\star(1,1,\alpha(p),\alpha(p))\alpha'(p) > 0.$$

and

$$\frac{dz_+(p)}{dp} \triangleq \frac{1}{t}\left(\frac{d\Lambda_2(p)}{dp} - \frac{d\Lambda_1(p)}{dp} - 1\right).$$

Evaluating this condition at $p^\star$, then we have

$$\mathcal{V}'(p^\star) = 2\underbrace{\left(\Lambda_2(p^\star) - t - \left(\Lambda_1(p^\star) - t\left(1-z_+^\star\right)\right)\right)}_{=p^\star}\frac{dz_+(p^\star)}{dp} + 2\left(1-z_+^\star\right)\frac{d\Lambda_1(p^\star)}{dp} + \left(2z_+^\star - 1\right)\frac{d\Lambda_2(p^\star)}{dp} - 2c'(\alpha^\star)\alpha'(p^\star).$$

Hence, defining $x_{11} = x^\star(1,1,\alpha(p),\alpha(p))$ and using the providers' first-order condition

$$\frac{1}{t}\frac{\partial\Lambda(\alpha_0,\alpha^\star)}{\partial\alpha_0}p_0 = c'(\alpha_0),$$

we have

$$\begin{aligned}\mathcal{V}'(p^\star) = {} & 2\frac{p^\star}{t}\left[\frac{2\alpha(p^\star)-1}{2}[f(x_{11}) - x_{11}]\alpha'^\star) + \frac{\mu\alpha(p^\star)}{2}x_{11}\alpha'^\star) - \frac{\mu}{2}x^\star(1,\alpha(p^\star))\alpha'(p^\star) - 1\right] + \\ & 2\left(1-z_+^\star\right)\frac{\mu}{2}x^\star(1,\alpha(p^\star))\alpha'(p^\star) + \left(2z_+^\star-1\right)\frac{2\alpha^\star(p^\star)-1}{2}[f(x_{11}) - x_{11}]\alpha'^\star) + \\ & \frac{\mu\alpha(p^\star)}{2}x_{11}\alpha'^\star) - 2\frac{p^\star}{t}\frac{d\Lambda(\alpha^\star,\alpha^\star)}{dp^\star} \\ = {} & 2\frac{p^\star}{t}\left[-\frac{\mu}{2}x^\star(1,\alpha(p^\star))\alpha'(p^\star) - 1\right] + \\ & 2\left(1-z_+^\star\right)\frac{\mu}{2}x^\star(1,\alpha(p^\star))\alpha'(p^\star) + \left(2z_+^\star-1\right)\frac{2\alpha^\star(p^\star)-1}{2}[f(x_{11}) - x_{11}]\alpha'^\star) + \frac{\mu\alpha(p^\star)}{2}x_{11}\alpha'^\star),\end{aligned}$$

which concludes the proof. ■

Proof of Proposition 8 Differentiating the value function $V(\alpha(p), N(p))$ with respect to p,

$$\begin{aligned}\frac{d\mathcal{V}(\alpha(p), N(p))}{dp} &= \left(\mu\frac{x^{\star}(1,\alpha(p)) - x^{\star}(0,\alpha(p))}{2} - N(p)\,c'(\alpha(p))\right)\alpha'(p) + \\ &\quad \frac{\phi\mu}{2(1-\phi)}\left[\frac{1}{\zeta(x^{\star}(1,\alpha(p)))} - \frac{1}{\zeta(x^{\star}(0,\alpha(p)))}\right]\alpha'(p) + \\ &\quad \frac{t}{4N(p)^2}N'(p) - N'(p)\left(c(\alpha(p)) + F\right).\end{aligned}$$

Evaluating at $p^{\star}$ the above condition, we then have

$$\frac{d\mathcal{V}(\alpha^{\star}, N^{\star})}{dp} = \frac{\phi\mu}{2(1-\phi)}\left[\frac{1}{\zeta(x^{\star}(1,\alpha^{\star}))} - \frac{1}{\zeta(x^{\star}(0,\alpha^{\star}))}\right]\alpha'(p^{\star}) + N'(p^{\star})\left(\frac{t}{4N^{\star 2}} - (c(\alpha^{\star}) + F)\right).$$

Using the zero profit condition $\frac{t}{N^{\star 2}} - c(\alpha^{\star}) - F = 0$, we then have

$$\frac{d\mathcal{V}(\alpha^{\star}, N^{\star})}{dp} = \frac{\phi\mu}{2(1-\phi)}\left[\frac{1}{\zeta(x^{\star}(1,\alpha^{\star}))} - \frac{1}{\zeta(x^{\star}(0,\alpha^{\star}))}\right]\alpha'(p^{\star}) - \frac{3t}{4}\frac{N'(p^{\star})}{N^{\star 2}},$$

which completes the proof. ■

Proof of Proposition 9. The equilibrium condition

$$\frac{\phi\mu}{4(1-\phi)}\left(\frac{1}{\zeta(x^{\star}(1,\alpha))} - \frac{1}{\zeta(x^{\star}(0,\alpha))}\right) = c'(\alpha^{*}),$$

can be obtained immediately from the providers' first-order condition. Hence, $\alpha^{\star} = \frac{1}{2}$ if and only if

$$\frac{1}{\zeta(x^{\star}(1,\alpha))} \leq \frac{1}{\zeta(x^{\star}(0,\alpha))},$$

which is always the case if $\zeta'(x) \geq 0$ for every $x \geq 0$, because $x^{\star}(1,\alpha) > x^{\star}(0,\alpha)$.

Consider now value maximization. As in the baseline model, expected value is

$$\mathcal{V}(\alpha) \triangleq \frac{1}{2}\sum_{s=0,1}\left[f(x^{\star}(s,\alpha)) - x^{\star}(s,\alpha)\right] + \frac{\mu}{2}\sum_{s=0,1}\Pr[s|\theta=1]\,x^{\star}(s,\alpha) - 2c(\alpha) - \frac{t}{12},$$

whose derivative evaluated at $\alpha^{\star}$ (i.e., substituting in the FOCs of E and L) yields

$$\mathcal{V}'(\alpha^{\star}) = \mu\frac{x^{\star}(1,\alpha^{\star}) - x^{\star}(0,\alpha^{\star})}{2} > 0,$$

which completes the proof. ■